Strategic Complementarities in Posted Wages*

Michael Droste†
Harvard University

April 1, 2024

Please click here for the latest version of this draft.

Abstract

Imperfectly competitive labor markets often exhibit strategic complementarity in wages: wage-setting decisions depend positively on other agents' wage-setting decisions. How important is this effect for macroeconomic dynamics? I develop a reduced-form empirical design to estimate the magnitude of strategic complementarity in posted wages using the near-universe of online job postings from 2010 to 2023. My design leverages *national wage-setters*, firms that do not vary posted wages across local labor markets, as a source of differential exposure to aggregate shocks. My preferred estimates indicate a modest degree of complementarity in posted wages: posted wages increase by 1% in response to a 10% increase in the mean posted wage of competitors. I assess the quantitative relevance of these results through the lens of a New Keynesian model with monopsonistic labor markets. Wage complementarities are a powerful source of real rigidity in this model; however, my empirical evidence suggests the effect of this channel is limited. My results suggest that strategic complementarity in wages is weaker than is often assumed, and provide an important "portable statistic" to discipline and evaluate models with imperfectly competitive labor markets.

*I am grateful to my advisors for their guidance and support: Gabriel Chodorow-Reich, Xavier Gabaix, and Ludwig Straub. I thank Robert Barro, Adrien Bilal, Niklas Flamang, Masao Fukui, Mark Kuperberg, Greg Mankiw, Ross Mattheis, Simon Mongey, Jesse Silbert, Jimmy Stratton, Stefanie Stantcheva, Christian Wolf, Lingxuan Wu, Eric Zwick, and seminar participants at Harvard for their helpful comments and suggestions. I also thank Bledi Taska, Layla O'Kane, and the data team at Lightcast Technologies for data and support.

†Email: mdroste@fas.harvard.edu.

1 Introduction

Imperfectly competitive labor markets exhibit strategic complementarity in wages when the wage-setting decisions of agents depend positively on the wage-setting decisions of other agents. This paper addresses two fundamental questions concerning this property. First, how strong are these wage complementarities in practice? Second, how important are wage complementarities for macroeconomic dynamics?

Strategic complementarities in product market price-setting have been an important element of business cycle theory since the seminal work of Ball and Romer (1990). Strategic complementarities in prices often manifest as a *real rigidity*, a mechanism that weakens firms' incentives to take actions that would stabilize output in response to shocks. New Keynesian models rely heavily on real rigidities in order to generate significant output volatility and persistent monetary non-neutrality under empirically plausible calibrations. A large research agenda has developed around strategic complementarities in *product market* price-setting, given the central role of product market competition and nominal rigidities in the New Keynesian paradigm. In this paper, I ask whether strategic complementarities in *labor market* wage-setting are empirically and theoretically relevant as a source of real rigidity and amplification mechanism in New Keynesian models.

Macroeconomic models that feature imperfectly competitive labor markets tend to exhibit strategic complementarity in wage-setting. For example, building off the seminal work of Blanchard (1986), quantitative macroeconomic models often assume monopolistically competitive labor markets in which households (or labor unions acting on behalf of households) set wages. This labor market structure is used in Smets and Wouters (2007) and a very large body of follow-up work that has been influential in monetary economics. In this class of model, wage complementarities arise primarily through the presence of non-constant labor demand elasticities. Similarly, labor markets characterized by search and matching frictions sometimes exhibit strategic complementarity in wages. Fukui (2022) shows that in a Burdett and Mortensen (1998) labor market with posted wages and on-the-job search, competition between firms over incumbent workers generates a novel strategic complementarity in wages that is stronger for more productive firms. Gertler and Trigari (2009) show that wage-setting complementarity can arise in a Diamond-Mortensen-Pissarides labor market with on-the-job search and staggered Nash bargaining.[1]

Although imperfectly competitive labor markets are an increasingly common element of macroeconomic models, there is relatively little empirical evidence on the

[1] Gertler and Trigari (2009) are careful to use the more generic term "wage spillover": when wages are determined by a bargaining rule, wage spillovers between firms are not *strategic*, in the sense of Bulow, Geanakoplos and Klemperer (1985).

magnitude of wage-setting complementarities in the real world. Staiger, Spetz and Phibbs (2010) study plausibly exogenous increases in the the wages of nurses at Department of Veterans Affairs hospitals in the U.S., and find modest (albeit imprecise) evidence that this led to increased wages for nurses at nearby non-VA hospitals. Derenoncourt et al. (2022) study the effects of voluntary minimum wage increases by specific national big-box retailers (e.g. Target) on the wages of competing retailers, and do not find do not evidence of an effect on competitors.[2] Both of these studies focus on a particular industry (nursing and retail services) at a particular point in time, potentially limiting the relevance of their empirical findings for the broader labor market. To the best of my knowledge, there is no direct reduced-form evidence on strategic interactions in wage-setting that expands beyond these particular industries.

The primary goal of this paper is to estimate the degree of strategic complementarity in *posted wages*, the wages listed by firms in online job advertisements. I begin by developing a novel exposure-based empirical design that I can implement with the universe of online job postings in the United States over the period 2010-2023. This design leverages spatial variation in the density of *national wage-setters*: firm-occupation pairs where posted wages do not vary across place within the United States. The intuition behind this design is simple. National wage-setters' wages cannot depend directly on place-specific factors, so local labor markets with relatively higher shares of national wage-setters are relatively more exposed to aggregate shocks. I construct an *exposure instrument* that can be interpreted as a local labor market's predicted differential exposure to aggregate shocks. The key identification assumption in this design is that exposure to national wage-setters in a given local labor market is conditionally exogenous with respect to other determinants of individual firms' posted wages. I use this instrument to estimate the *posted wage elasticity*, the elasticity of a firm's posted wage with respect to the mean wage of competing firms.

My estimates suggest that elasticity of posted wages with respect to the mean posted wage of competitors is between 0.05 and 0.20. In other words, I find that a firms' posted wage increases by 0.5% to 2.0% in response to a 10% exogenous increase in the mean posted wage of competing firms. I do not find evidence that wage-setting complementarities vary with local labor market concentration or firm size. I find modest evidence that complementarities vary across NAICS sectors: manufacturing, construction, and mining exhibit relatively weaker elasticities, while financial activities and information exhibit relatively stronger elasticities. Firms that have historically paid relatively high wages exhibit relatively stronger responses. My estimates are robust to a wide variety of alternative plausible specifications, including

[2]An early draft of Derenoncourt et al. (2022) reported substantial spillover effects. A revised draft documents that mean reversion in the dependent variable led to spurious significant estimates under their empirical design. Using a revised design that is robust to mean reversion, a revised draft of this paper cannot reject a null of no effect.

alternative occupational and geographic definitions, sample weights, and definitions of national wage-setters. These results are also robust to specifications that explicitly allow for dynamic or lagged adjustment in posted wages. Notably, my estimates are significantly smaller than comparable evidence on strategic complementarity in goods pricing, as documented by Amiti, Itskhoki and Konings (2019).

Because my reduced-form approach does not require taking a stand on any specific structural mechanisms that generate strategic interactions in wage-setting, my estimates are a "portable statistic" (in the sense of Nakamura and Steinsson (2018)) that can be used to discipline a very broad class of models with imperfectly competitive labor markets. I illustrate this important point by comparing my estimates with the implied degree of wage complementarity in several common settings. For instance, in a monopsonistic labor market with firm-specific CES labor supply and decreasing returns to labor, the degree of strategic complementarity in wage-setting is pinned down by the elasticity of output with respect to labor and the firm-specific labor supply elasticity. For instance, Mui (2021) develops a New Keynesian model with monopsonistic labor markets and CES labor supply. This calibrated model implies a wage-setting elasticity (with respect to the relevant wage index) of 0.248 – modestly larger than my baseline. In contrast, "medium-scale" DSGEs in the vein of Smets and Wouters (2007), which use Kimball (1995) aggregators over labor varieties to generate significant complementarity in wage-setting, typically rely on calibrations with a high degree of "Kimball curvature" – such calibrations often imply a relatively high degree of wage complementarity.

In the final part of this paper, I consider the quantitative relevance of my reduced-form estimates in the context of a New Keynesian model with monopsonistic labor markets. The structure of this model preserves much of the analytical simplicity and intuition of the canonical New Keynesian framework: household consumption/savings behavior is summarized by an Euler equation, and a New Keynesian Phillips curve dictates the relationship between output and inflation. Relative to this standard benchmark, this model features a monopsonistic labor market with firm-specific Kimball labor supply. This framework nests both a monopsonistic labor market with CES labor supply and a competitive labor market as special cases, and can flexibly accommodate any degree of strategic complementarity (or substitutability) in wage-setting with three parameters. Importantly, this flexibility allows me to distinguish between the real rigidity that emerges due to monopsony power from strategic complementarity in wages. In this model, "micro complementarities" make real wages relatively less responsive to nominal shocks, increasing the volatility of output and amplifying the degree of monetary non-neutrality exhibited by the model.

This paper contributes to several strands of literature in macroeconomics and labor economics. My primary contribution is the direct measurement of strategic complementarity in posted wages with reduced-form empirical methods. This contributes to an existing body of empirical research studying strategic complementarity in other price-setting environments. Perhaps the best empirical evidence on the magnitude of price-setting complementarities comes from Amiti, Itskhoki and Konings (2019), who likewise use IV methods to trace out quasi-exogenous shocks to competing firms' prices. In comparison to my estimates, these authors find significantly higher elasticity of firm prices with respect to their competitors' prices (about 0.35). For labor markets in particular, my empirical results build on past work by Staiger, Spetz and Phibbs (2010) and Derenoncourt et al. (2022), who both focus on studying wage spillovers resulting from wage changes at a small number of firms. Relative to this existing work, I focus on a much broader set of firms and industries. In addition to providing these estimates, the empirical design that I develop has the potential to be leveraged in other environments as well. For instance, Dellavigna and Gentzkow (2019) document that uniform pricing across locations is relatively common for certain retailers. My exposure design can potentially be applied to this setting with retail price microdata.

I also contribute to a broader literature in macroeconomics that studies potential sources of real rigidity in New Keynesian models. An important paper by Golosov and Lucas (2007) argued that simple New Keynesian models with menu costs are unable to generate nontrivial and persistent monetary non-neutrality under calibrations informed by the available micro evidence on price adjustment for goods. This critique has led to a resurgent interest in amplification mechanisms and real rigidities in New Keynesian models. For example, Nakamura and Steinsson (2010) demonstrate that trade in intermediate inputs ("roundabout production") and sticky prices interact to amplify the degree of monetary non-neutrality exhibited by menu cost models without requiring calibrations at odds with micro evidence on goods pricing. Earlier work by Kimball (1995) demonstrated that variable product markdowns can generate a substantial amount of real rigidity; the Kimball aggregator is now commonplace in large-scale New Keynesian models, as in Smets and Wouters (2007). Against the backdrop of this extensive literature on imperfectly competitive markets, this paper seeks to provide direct reduced-form empirical evidence on the strength of complementarities in labor markets to assess whether these complementarities can generate a realistic degree of real rigidity.

The remainder of this paper proceeds as follows. Section 2 describes a simple static monopsonistic labor market with strategic complementarity in the wage-setting decision of firms. I demonstrate how strategic complementarity can arise in one particular setting, and use this model to motivate a reduced-form estimating equation relating a firm's posted wage to the average posted wage of their competitors. Section 3 contains my empirical analysis. This section describes my data and empirical setting, develops an exposure-based instrumental variables strategy, and reports my baseline estimates and a variety of associated tests for heterogeneity and robustness. Section 4 embeds the stylized monopsonistic labor markets of Section 2 in an otherwise standard New Keynesian model. I use this model to explore how strategic complementarities in wage-setting are relevant for macroeconomic dynamics. Section 5 concludes by reviewing the key contributions of this paper and describing some avenues for future research.

2 Strategic Complementarities in Wage-Setting

I begin with a simple motivating example to illustrate strategic interactions in wage-setting. This example will allow me to precisely define strategic complementarity in wage-setting, provide intuition on how it can arise, and motivate an estimating equation for my empirical analysis. Because this model is relatively simple, I defer the derivations of the results I present to Appendix B.1.

The economy is populated by a representative household and a unit mass of atomistic firms indexed by i. The household consumes a homogeneous consumption good and supplies firm-specific labor to each firm i. Firms produce and sell a homogeneous output good to households using firm-specific labor ℓ_i. The output market is competitive, and the labor market is monopsonistically competitive, as each firm i is the sole source of demand for a given variety of labor.

The representative household chooses consumption and labor supply to maximize utility, subject to their budget constraint and a Kimball aggregator implicitly defining L from varieties $\{\ell_i\}$. The household's utility maximization problem is:

$$\max_{C,L,\{\ell_i\}_i} \quad u(C,L) \quad \text{s.t.} \quad C \leq \int_0^1 w_i \ell_i di \quad \text{and} \quad 1 = \int_0^1 \Upsilon(\ell_i/L) di$$

where the utility function u is assumed to be twice differentiable, strictly concave increasing in C, and strictly convex decreasing in L. The household takes the price of the consumption good and wages $\{w_i\}$ as given, where I have chosen the consumption good as the numeraire and normalized its price to one.

The constraint $1 = \int \Upsilon(\ell_i/L)di$ is a Kimball (1995) aggregator that implicitly defines aggregate labor supply L from the bundle of firm-specific labor supply $\{\ell_i\}$. The function Υ is strictly increasing and twice differentiable, with $\Upsilon(0) = 0$. Intuitively, the function $\Upsilon(\cdot)$ defines weights on each element of the bundle $\{\ell_i\}_i$ in the mapping $\{\ell_i\}_i \to L$. For instance, if $\Upsilon(\cdot)$ is a power function, L is a CES aggregate over varieties ℓ_i. I will return to characterizing Υ shortly.

Utility maximization gives rise to upward-sloping firm-specific labor supply curves:

$$\frac{w_i}{W} = \Upsilon'(\ell_i/L) \qquad \text{where} \quad W = \frac{\int_0^1 w_i(\ell_i/L)di}{\int_0^1 \Upsilon'(\ell_i/L)(\ell_i/L)di}$$

where the wage index W depends on Υ, and will only correspond to an "ideal" welfare-relevant wage index when firm-specific labor supply elasticities are constant (e.g. CES). The firm-specific (own-wage) labor supply elasticity, ϵ_i, can be written:

$$\epsilon_i(\ell_i/L) \equiv \frac{\partial \ln \ell_i}{\partial \ln w_i} = \frac{\Upsilon'(\ell_i/L)}{\Upsilon''(\ell_i/L)(\ell_i/L)}$$

It will be helpful to impose a functional form on $\Upsilon(\cdot)$. I consider the following form by Dotsey and King (2005), adapted to labor supply:

$$\Upsilon(\ell_i/L) = \frac{\omega}{1+\psi}\Big[(1+\psi)(\ell_i/L) - \psi\Big]^{1/\omega}$$

where ω is a composite parameter defined by $\omega \equiv (1+\psi)\phi/(1+\phi\psi)$. The two parameters in this system are $\phi \in (0,1)$, which characterizes the elasticity of firm-specific labor supply at $w_i = W$, and ψ, which characterizes the curvature of the firm-specific labor supply curve.

Given this functional form, we can express relative labor supply for firm i as:

$$\ell_i/L = \frac{(w_i/W)^{\omega/(1-\omega)} + \psi}{1+\psi}$$

where it is useful to note that when $\psi = 0$, labor supply collapses into a standard "CES-type" labor supply system. I will consider this special case as an important benchmark in my analysis to follow.

Figure 1 depicts firm-specific log relative labor supply under several choices of the Kimball curvature parameter ψ. In each case, I set $\phi = 0.5$, which implies a unit elasticity of firm-specific labor supply at $w_i = W$. The black line depicts the benchmark CES case with $\psi = 0$. The remaining lines plot labor supply under $\psi \in \{-6, -3, 3, 6\}$. Intuitively, the "Kimball curvature" parameter ψ characterizes the departure of firm-specific labor supply from a CES benchmark: $\psi < 0$ corresponds to log-convex firm-specific labor supply (ϵ_i decreasing in w), and $\psi > 0$ corresponds to log-concave firm-specific labor supply. This figure illustrates why the Kimball system is attractive: it retains much of the analytical convenience of CES-type labor supply, nesting it as a special case, while parsimoniously tying the curvature of log labor supply to a single parameter.

I now turn to analyzing the wage-setting decision of firms. There are a unit mass of atomistic firms indexed by i. Each firm i is the sole purchaser of a given labor variety i supplied by the household. Firm i produces a homogeneous output good with the production function $F(\ell_i) = A_i \ell_i^{1-\alpha}$, where $1-\alpha$ indicates the elasticity of output with respect to labor and A_i denotes firm-specific productivity. Output is sold competitively; I normalize the price of the output good to one.

Each firm i chooses a real wage w_i to maximize real profits, subject to firm-specific labor supply provided by the household, taking W and L as given. The firm's static profit maximization problem is:

$$\max_{w_i} \quad A_i \ell_i^{1-\alpha} - w_i \ell_i \qquad \text{s.t.} \quad \ell_i / L = \frac{(w_i/W)^{\omega/(1-\omega)} + \psi}{1+\psi}$$

The Kimball labor supply system retains two key elements from a Dixit-Stiglitz-type CES labor supply system that allows it to be extremely tractable. First, firm-specific labor supply depends only on *relative* wages. Second, because each firm is atomistic, their decision has no impact on the aggregate wage index, which they take as given. That is, firms compete in wages against a single *aggregate* wage index, and each firm is too small for their decisions to impact aggregates. The appropriate way to think about strategic interactions in wage-setting in this environment is between an individual firm's wage w_i and the aggregate wage W against which they compete for labor. The following definition makes this concept precise:

Definition 1 *Let w_i^* denote the wage that solves the firm's profit-maximization problem, subject to firm-specific labor supply and taking the aggregate wage index W as given. Wages are said to be strategic complements if $\partial w_i^* / \partial W > 0$, strategic substitutes if $\partial w_i^* / \partial W < 0$, and strategically independent if $\partial w_i^* / \partial W = 0$ for all i.*

Intuitively, when wages are strategic complements, an increase in the aggregate wage causes firms to raise their own wage.[3] For ease of exposition, in the remainder of this paper I adopt the term 'wage complementarity' to refer to strategic complementarity in wage-setting, and 'wage interaction' to refer to strategic interactions in wage-setting more broadly.

The optimal wage w_i^* can be expressed in "Lerner form" as a gross markdown $\mathcal{M}_i$ on the marginal product of labor $F_{\ell i}$, $w_i^* = \mathcal{M}_i \times F_{\ell i}$. This representation of the wage provides a convenient lens to think about where wage complementarities can arise:

$$\frac{\partial w_i^*}{W} = \frac{\partial \mathcal{M}_i}{\partial W} F_{\ell i} + \frac{\partial F_{\ell i}}{\partial W} \mathcal{M}_i$$

[3] A more general definition of strategic complementarity in line with Bulow, Geanakoplos and Klemperer (1985) is that the firm's value function is supermodular in (w_i^*, W). These are equivalent when $\partial^2 \pi / \partial w_i^* \partial W$ is defined everywhere, which is generically true in this model.

Intuitively, wage complementarities can only arise in this model if the wage index W has a direct effect on the gross markdown or the marginal product of labor. In particular, as $\mathcal{M}_i$ and $F_{\ell i}$ are positive, wage complementarity can only happen if the markdown and/or the marginal product is increasing in W. In practice, many imperfectly competitive labor markets commonly analyzed in the literature rule out one or both of these channels if the markdown and/or marginal product is constant (or at least independent of W) in equilibrium. This expression also illustrates why we have gone through the trouble of defining labor supply in terms of a generalized Kimball aggregator, rather than the more commonplace CES special case: with CES labor supply, the wage markdown is constant, restricting one of these two channels immediately. It is useful to illustrate how each complementarities arise through each channel by considering two cases.

I start by considering the case where the firm-specific labor supply elasticity is constant, and the "Kimball curvature" parameter $\psi = 0$. In this case, the gross wage markdown is constant, and wage complementarity can only arise through a non-constant marginal product of labor. The following proposition characterizes wage complementarities in this CES environment:

Proposition 1 *Suppose that $\psi = 0$. Let $\epsilon_i = \partial \ln \ell_i / \partial \ln w_i^* = \phi/(1-\phi)$ denote the firm-specific (own-wage) elasticity of labor supply. The optimal wage w_i^* satisfies:*

$$\ln(w_i^*) = \kappa + \beta \ln(W) + (1-\beta)\ln(A_i) \tag{1}$$

where $\beta = \frac{\alpha\epsilon_i}{1+\alpha\epsilon_i}$ denotes the elasticity of the firm's optimal wage w_i^* with respect to the aggregate wage index W.

Notice that when $\alpha = 0$, the marginal product of labor is constant, and wage-setting exhibits strategic independence, $\beta = 0$. This is an important knife-edge case because it sharply distinguishes monopsony power from strategic complementarity. More generally, with a decreasing (increasing) marginal product of labor, wages are strategic complements (substitutes), and the strength of this complementarity (substitutability) is decreasing in monopsony power (as measured by the inverse firm-specific labor supply elasticity or the gross markup). Thus, when labor supply exhibits a CES form, strategic interactions in wage-setting are determined only by α and ϵ_i, and can be characterized by the *wage complementarity elasticity* $\partial \ln w_i^* / \partial \ln W$.

Next, I consider the opposing case where wage-setting complementarities can only arise through variable markdowns. Suppose that $\alpha = 0$, so that the marginal product of labor is constant (independent of W). In this case, wage-setting complementarities emerge through variable gross wage markdowns, which in turn reflect non-constant labor supply elasticities. The following proposition characterizes the elasticity of the optimal wage w_i^* with respect to the aggregate wage index W in terms of the own-wage labor supply elasticity:

Proposition 2 *Let $\Pi(w_i, W)$ denote real firm profits as a function of the wage w_i and the aggregate wage index W, and let w_i^* denote the wage satisfying the first-order condition $\Pi_1(w_i^*, W) = 0$. Denote the the firm-specific (own-wage) labor supply elasticity as $\epsilon_i \equiv \partial \ln \ell_i / \partial \ln w_i^*$. When $\alpha = 0$, the elasticity of w_i^* with respect to W at a symmetric equilibrium $w_i = W$ is:*

$$\frac{\partial \ln w_i^*}{\partial \ln W} = \frac{\ell_i}{-\Pi_{11}} \frac{-\partial \epsilon_i / \partial w_i^*}{\epsilon_i}$$

This proposition immediately implies that $\partial w_i^* / \partial W = 0$ under CES labor supply, as the own-wage elasticity is constant. Moreover, if the own-wage labor supply elasticity ϵ_i is decreasing in w_i - which is the case when $\psi < 0$, as shown in Figure 1 - then wage-setting exhibits strategic complementarity. Otherwise, if $\partial \epsilon_{\ell i} / \partial w_i < 0$, wage-setting exhibits strategic substitution. The proof for this proposition is relegated to Appendix B.1.3.

In both of these examples, the elasticity of wages with respect to the aggregate wage index, $\partial \ln w_i^* / \partial \ln W$, characterizes the magnitude of strategic interaction in wage-setting. For this reason, my empirical analysis will focus on estimating this elasticity, which I will refer to as the *wage-setting* or *wage-posting* elasticity.

While this static model environment captures two mechanisms that can generate strategic complementarity in wages, it does not exhaustively characterize all possible mechanisms that can generate wage complementarity. When wage-setting is dynamic (for instance, due to nominal rigidities in wage-setting, e.g. Calvo sticky wages), labor markets do not clear (e.g. search and matching frictions), or when individual agent's wage-setting decisions impact aggregates (e.g. oligopsony and granular firms), wage-setting decisions are more complex. As Alvarez, Lippi and Souganidis (2022) note, strategic complementarities in price-setting environments are extremely difficult to describe generally. The objective of this section is merely to motivate wage-setting complementarity in a simple environment, and to motivate the estimand $\partial \ln w_i^* / \partial \ln W$. In my empirical analysis in Section 3, I will not rule out any of these alternative channels.

This model has two related properties worth discussing in more detail before proceeding. First, monopsony power - upward-sloping firm-specific labor supply curves - are *not sufficient* to imply strategic interaction between firms in wage-setting. That is, monopsonistic labor markets need not exhibit strategic interactions in wages. In particular, CES labor supply ($\psi = 0$) and a constant marginal product of labor ($\alpha = 0$) implies strategic independence in wage-setting *for any* firm-specific labor supply elasticity. Second, although $|\beta|$ is generically increasing in the firm-specific labor supply elasticity, *any* amount of strategic complementarity $|B|$ is compatible with any finite labor supply elasticity. Indeed, if firm-specific labor supply is log-concave

($\psi > 0$), or if the marginal product of labor is increasing in ℓ_i, this model can also generate an arbitrary degree of strategic *substitutability* in wages.

The next section proceeds with my empirical analysis, where I aim to estimate an empirical counterpart to the elasticity $\partial \ln w_i / \partial \ln W$ with observational data. In the remainder of this paper, my focus is *not* on disentangling mechanisms that generate strategic complementarities in wage-setting. My empirical design does not rely on, or isolate, any particular channel (e.g. variable markdowns or marginal products). The primary objective of my empirical analysis is to provide a reduced-form estimate of β with an empirical design motivated by the loglinear pricing equation (1).

3 Measuring Complementarities in Posted Wages

I now turn to the core contribution of this paper, estimating the degree of strategic complementarity in posted wages in the United States. This section describes my data and sample construction, my empirical methodology, my preferred estimates, and a variety of associated extensions and robustness tests.

3.1 Data and Sample

My analysis primarily relies on a dataset comprising the near-universe of online job postings in the United States from January 2010 to July 2023, collected by Lightcast (formerly known as Burning Glass Technologies). Lightcast scrapes job postings from over 65,000 online and print sources, with no single source accounting for more than 5% of postings in the data. This data is thought to comprise around 70% of posted vacancies in the United States (Hazell et al. (2022)). I supplement this posting-level data with information on job openings and hires by sector and state from the Job Openings and Labor Turnover Survey (JOLTS), a product of the Bureau of Labor Statistics (BLS).

Figure 2 compares job openings in Lightcast against JOLTS job openings data over the period 2010-2023. The underlying notions of job openings vary slightly across these two sources. Job openings in JOLTS correspond to the stock of unfilled job openings at the end of each month. In contrast, the Lightcast job openings are obtained by summing new job postings in each month. To bring these concepts closer together, this figure plots the difference between job openings in months t and $t-1$ plus new hires in month t in JOLTS, and compare this against total new job postings in Lightcast each month. In JOLTS, monthly job openings have grown from approximately 4 million/month to around 6 million/month over the period 2010-2023, with notable fluctuations particularly during COVID. In contrast, Lightcast postings have grown from about 1 million/month to 3.5 million/month over this period. The relative growth in Lightcast postings as a share of JOLTS job openings likely reflects a combination of increased coverage in the online sources scraped by Lightcast, particularly over the period 2010-2013, and a broader shift toward online job postings in the labor market over this time period.

Each observation in the Lightcast microdata corresponds to a single job posting. Lightcast de-duplicates postings across sources to ensure that the same job posting scraped from multiple sources is not represented in the data multiple times.[4] Job postings typically include the posting date, job title, job location, posted wage/salary information (either a "point" wage/salary offer or a range), job requirements (e.g. years of experience in related industries; educational/degree/certification requirements), pay structure (e.g. part-time/full-time; hourly/salaried), and the employer. Wage/salary information is annualized by Lightcast by converting the salary into a full-time equivalent, assuming 40 hours/week and 50 weeks/year for non-salaried jobs. Annualized wage/salary information is top-coded at $500,000 by Lightcast; this accounts for a negligible share of postings.[5] Lightcast maps job titles and employers to standard occupational classification and industrial classification systems. Each employer is associated with an industry using the (six-digit) North American Industrial Classification System (NAICS), and each job title is assigned an occupation according to the Standard Occupational Classification (SOC) system and the closely-related O*NET-SOC occupational classification system.

Most online job postings do not contain wage/salary information. Approximately 20% of job postings in the Lightcast data over the period 2010-2023 include some information on wages or salaries. Posted wage/salary information has become somewhat more common over time, particularly after the COVID pandemic. Figure 3 documents the share of postings in the Lightcast data with wage information over time. About 15% of job postings included wages from 2010-2018; over the last five years, this number has increased dramatically. In 2022, nearly 40% of job postings contained wage or salary information.[6] I follow Hazell et al. (2022) and only use advertisements with 'point' wage/salary information provided, rather than those posting a range.

Very recent work by Batra, Michaud and Mongey (2023) argues that the online job postings by Lightcast are contaminated by *imputed* wages by job boards. That is, these authors argue that a significant fraction of job postings do not contain the true wage offers by firms, but rather an estimate that is provided by job boards -

[4]Details concerning this de-duplication procedure are described in Data Appendix A

[5]24,357 postings have annualized wages top-coded at $500,000; this corresponds to less than 0.1% of postings in my baseline sample.

[6]Recent legislation may have influenced the increase in wage information. For instance, starting in 2023, employers with more than 15 employees in the state of California are *required* to provide wage/salary bands for all online job postings.

analogous, for instance, to home price estimates on home price sites (e.g. Zillow). Ideally, my analysis would exclude imputed wages entirely from the dataset; however, Lightcast does not collect information on whether wages are imputed. I will address this critique in two ways. First, I consider the 2010-2017 subsample - that is, excluding all postings from 2018 to 2023. This sample conservatively excludes *all* observations in the problematic 2018-2023 sample window identified by Batra, Michaud and Mongey (2023). Of course, this is a rather crude way to handle this issue, because it excludes many postings in 2018-2023 in which wages are not imputed. This motivates an alternative subsample that attempts to more systematically exclude likely imputed wages from postings in the years 2018-2023. Specifically, I also consider a subsample in which I exclude all postings from job boards in which the prevalence of wage/salary information more than doubles over the period 2018-2023, relative to the rate of nonmissing wage/salary information over the period 2010-2017. I defer a more detailed discussion of these approaches to Section 3.5.

I produce a monthly panel with firm-occupation-place units from the raw Lightcast microdata as follows. First, I define the *job* (indexed by j) associated with a given posting as a combination of employment type (full time or part time) and the posting's SOC 5-digit ("broad occupation") occupation code. Next, I define a posting's *location* (indexed by l) as the commuting zone associated with a given posting. Commuting zones are agglomerations of U.S. counties, defined from county labor flows, that approximate local labor markets (see Autor and Dorn (2013)). Most commuting zones are comprised of three to five U.S. counties. Next, I aggregate the posting-level microdata to produce a panel at monthly frequency, where each observation corresponds to a firm-job-place (ijl) in month t. My baseline sample comprises all firms with nonmissing posted wage/salary information, excluding large job recruitment agencies, firms without geographic or occupational classifications, and internships.

The key dependent variable in my empirical analysis is w_{ijlt}, the log average wage posted by firm i for job j in location l and month t. The key dependent variable, treated as the endogenous regressor in my IV design, is $\bar{w}_{jlt}$: the log mean posted wage in job j, location l, and time t. I construct $\bar{w}_{jlt}$ by taking the (posting-weighted) average of posted wages in a given (j, l) cell in months $t-3$, $t-2$, and $t-1$. This implies that the dependent variable (dated t) does not enter directly $\bar{w}$, which depends on posted wages in the preceding months. These timing restrictions are *ad hoc*, necessary, and potentially nontrivial for my analysis. I will therefore revisit this definition in my robustness checks, specifically by considering alternative specifications with alternative windows over which competitors' posted wages are aggregated (e.g. 4 months, 6 months).

3.2 National Wage Setters

My empirical strategy requires identifying which job postings are associated with *national wage setters*. National wage-setters are firm-occupation pairs that do not vary posted wages systematically across commuting zones within the United States. In a sense, national wage-setting is the labor market analog of uniform pricing among retailers, studied by Dellavigna and Gentzkow (2019). My work here builds off Hazell et al. (2022), who define the term "national wage-setter" and document a number of facts concerning the distribution of national wage-setters across the U.S. using the same Lightcast data. My empirical design, outlined in more detail in Section 3.3, leverages variation in the share of national wage-setters (across local labor markets and over time) as a source of quasi-exogenous exposure to aggregate shocks.

Identifying national wage-setters from observational data is nontrivial due to the sparsity of job postings within firm-occupation across commuting zones and over time. In my baseline sample, approximately one-quarter of observations correspond to firms that post less than 10 vacancies total across place over the sample period 2010-2023. For these firms, it is difficult to distinguish between temporal and spatial variation in wages. It is helpful to consider a minimal (fictitious) example to illustrate this concern. Suppose that a firm hires full-time salaried medical technicians in two commuting zones, A and B. We observe five postings in A with an annual salary of \$52,000 in 2019, and three postings in B with an annual salary of \$55,000 in 2020. From this information alone, it is impossible to tell whether this observed difference in posted wages reflects variation across commuting zones, variation over time, or both. I address this issue by using a conservative definition of national wage-setter that is unlikely to misclassify firms that *may* vary wages across places as national wage-setters:

Definition 2 *A firm-job pair ij is a national wage-setter in year $y(t)$ if: (i) There are job openings associated with ij in at least three commuting zones in year $y(t)$; (ii) ij has postings in more than one commuting zone in at least two quarters of year $y(t)$; (iii) the modal posted wage for firm-job ij does not vary across commuting zones within each quarter of the calendar year.*

Intuitively, we define a firm-job pair as a national wage-setter if the modal posted wage for this firm-job pair does not vary across commuting zones within calendar year, and we observe this firm-job pair in at least three commuting zones and multiple quarters in a calendar year. I follow Hazell et al. (2022) by using 6-digit SOC codes to classify occupations for the purposes of defining national wage-setters.[7]

This definition deviates from the concept explored by Hazell et al. (2022) in a few respects. First, Hazell et al. use counties as geographic units, while I rely on

[7] I have also explored defining national wage-setters using both O*NET and SOC-5 occupational classifications. My results are robust to these alternative occupational definitions.

commuting zones. Commuting zones are constructed by agglomerating neighboring counties based on commuting flows, so commuting zones are a (weakly) coarser geographic entity. Relative to Hazell et al. (2022), I do not allow firm-job pairs that are present in only one or two commuting zones in a given year to be considered national wage-setters. I impose this restriction to reduce the occurrence of national wage-setters who operate in a very constrained geographic region, a concern I will revisit in more detail later. It turns out that this restriction is not particularly strong: a large majority of national wage-setters operate in more than 50 commuting zones, and span many states. Later, I will consider an alternative and stronger definition of national wage-setters that requires firms to operate in many commuting zones, and demonstrate that my results are robust to this alternative definition. Lastly, I exclude any firm-job pair from being identified as a national wage-setter if they do not post vacancies in at least two quarters in a calendar year. I impose this restriction so that it is easier to distinguish variation in posted wages for a given firm-job pair over time and across commuting zones, as in the simple example involving medical technicians previously described.

Table 1 provides summary statistics for my analysis sample. Panel A corresponds to the full sample, while Panels B and C correspond to the subsamples comprising all national wage-setters and local (non-national) wage-setters, respectively. The average posted wage (expressed as full-time salary equivalent) in this panel is \$60,600 in 2020 dollars. About 80% of postings correspond to full-time job postings, and about 33% of postings with educational information specified require a bachelor's degree or above. The prevailing mean wage $\bar{w}$ associated with a given posting is computed from an average of 1342 postings. Relative to this baseline, job postings by national wage-setters have higher posted wages on average (\$77,000), are more likely to require a college degree, and are more frequently posted in smaller local labor markets with fewer competitors.

Figure 4 depicts the share of job postings by national wage setters across commuting zones, averaged over all occupations and over the entire sample period. In the baseline sample, approximately 4% of postings correspond to national wage-setters. However, there is considerable variation across commuting zones. In the bottom 10% of commuting zones (unweighted), national wage-setters account for fewer than 2.2% of postings, while the share of posts by national wage-setters exceeds 6.0% in the top 10% of commuting zones. There are obvious regional patterns evident in the graph: the west coast and much of the eastern seaboard exhibit low rates of national wage-setting, while much of the midwest and south exhibit relatively high rates. However, there is substantial variation across commuting zones within states: about two-thirds of the variation in national wage-setting shares across commuting zones is within-state.

3.3 Empirical Methodology

Motivated by the loglinear CES wage-setting equation (1) from Section 2, I consider a reduced-form empirical specification that relates the log posted wage associated with a given job posting (w_{ijlt}) to the log mean posted wage in the same occupation and commuting zone over the preceding three months ($\bar{w}_{jlt}$):

$$w_{ijlt} = \lambda D_{ijlt} + \beta \bar{w}_{jlt} + \epsilon_{ijlt} \tag{2}$$

where D_{ijlt} denotes a vector of controls (which can include fixed effects), λ denotes a vector of coefficients, and ϵ is a mean zero structural residual. The parameter of interest, β, corresponds to the elasticity of posted wages with respect to the posted wages of competing firms in the same local labor market.

OLS estimation of (2) with observational job postings data will not generally yield a consistent estimate of the population parameter β. Unobserved common shocks will induce comovement between a given wage and the average wage of competitors, even in the absence of strategic interactions between firms. A vector of controls and fixed effects in (2) cannot address all possible common shocks or confounds. I resolve this problem by constructing an instrumental variable that can be interpreted as quasi-random exposure to aggregate shocks across local labor markets.

This IV design is motivated by the observation that national wage-setters' wages cannot depend directly on location-specific factors. Variation in exposure to national wage-setters across local labor markets therefore corresponds to variation in exposure to aggregate shocks. I construct an instrument for $\bar{w}$ in (2) by interacting a given local labor market's share of national wage-setters s_{jlt} with the log average posted wage of national wage-setters in the same labor market, $\bar{w}^N_{jlt}$. If the share of national wage-setters in a given local labor market is orthogonal to other determinants of posted wages (e.g. place-specific shocks), this instrument isolates a source of variation that can be used to consistently estimate β.

A Simple Factor Model Example

I will take a brief detour to consider a stylized model to illustrate concretely how this instrument can provide a useful source of identifying variation in $\bar{w}$. Let w^N_{jlt} and w^L_{jlt} denote the log mean posted wage of national wage-setters and local (i.e. non-national) wage-setters, respectively, for job j, location l, and time t. Let s_{jlt} denote the share of job postings from national wage-setters in local labor market jl at time t. Suppose that log mean wages of local wage setters can be expressed as a linear combination of shocks and the mean wage of competing firms:

$$w^L_{jlt} = \alpha^L_{jlt} + \lambda_L X_{jt} + Y_{jlt} + \beta_L \bar{w}_{jlt}$$
$$w^N_{jlt} = \alpha^L_{jlt} + \lambda_N X_{jt} + \beta_N \bar{w}_{jt} + \epsilon^N_{jt}$$

where α^L_{jlt} and α^L_{jlt} denote intercept terms; X_{jt} and Y_{jlt} represent vectors of aggregate and place-specific shocks, respectively; λ_L and λ_N represent vectors of aggregate factor loadings, normalizing the factor loading on Y to 1; β_L and β_N represent the responsiveness of firm i wages to competing firms, and $\bar{w}_x$ denotes the log average wage of all firms in x.

I make two simplifying assumptions in the interest of exposition. First, I assume that $\beta_N = \beta_L$. This is a minor assumption made purely to simplify the algebra in this stylized example. Second, I assume that local labor market shocks Y_{jlt} do not enter the average wages of national wage-setters, w^N_{jlt}, even indirectly (i.e. summing over all places). In principle, this excludes the plausible scenario in which national wage-setters instead respond to some weighted average of place-specific shocks to which they are exposed:

$$w^N_{ijlt} = \gamma_{ijl} + \lambda_N X_{jt} + (|L_{ijt}|)^{-1} \sum_{s \in L(ijt)} \theta_{ijs} Y_{jst}$$

where γ_{ijl} denotes a unit (firm-job-place) intercept term, θ represents firm i's factor loading on local labor market shocks and L_{ijt} denotes the set of locations that firm-job ij operates in period t. As the sum of place-specific shocks on the right-hand side does not directly depend on location l, this equation is consistent with national wage-setting. If L is large and the place-specific shocks Y are i.i.d. across places and time, a law of large numbers argument implies that the contribution of place-specific shocks vanishes with the root of L.

I omit this complexity merely to simplify the exposition of this simple motivating example for my IV strategy. However, this concern is potentially a problem for my empirical strategy, which relies on the assumption that the average wages of national wage-setters do not reflect aggregate shocks. First, some national wage-setters might operate in relatively few local labor markets, so that their wage may depend on the sum of relatively few local labor market shocks that may not "wash out" by the law of large numbers. Second, local labor market shocks may exhibit spatial correlation. Under these conditions, local labor market shocks will not generally vanish from the local average $\bar{w}^N_{jlt}$. I will defer a detailed discussion of this to Section 3.5.

I now return to the example. Using the (approximate in logs) decomposition $\bar{w}_{jlt} \approx s_{jlt}\bar{w}^N_{jlt} + (1 - s_{jlt})\bar{w}^L_{jlt}$, the average wage of national wage-setters can be expressed purely in terms of aggregate factors X_{jt} and the national share of wage-setters s_t:

$$\bar{w}_{jlt}^N = \left[\lambda_N + \frac{\beta}{1-\beta}\Big(\lambda_L + s_t(\lambda_N - \lambda_L)\Big)\right] X_{jt} \tag{3}$$

Note that the in the case with uniform aggregate factor loadings $\lambda_N = \lambda_L = \lambda$, the right-hand side of this equation collapses to $\frac{\lambda}{1-\beta}X_{jt}$. Intuitively, if the factor loading on the aggregate factor is the same for national and local wage-setters, the share of national wage-setters is irrelevant. Importantly, note that the expression above does not depend on the local shocks Y_{jlt}, or more generally any location-specific factor. That is, $\bar{w}_{jlt}^N$, the log average wage of national wage-setters, does not depend on l.

Suppose that we observe individual (log) wages w_{ijlt}, national wage-setter shares s_{jlt}, and (log) average wages $\bar{w}$, $\bar{w}^N$, and $\bar{w}^L$. We wish to estimate the structural parameter β from the structural model (2). Assume that the vector of controls D includes flexible time-occupation, time-location, and firm-occupation-location fixed effects. The location-specific factors Y_{jlt} contaminate $\hat{\beta}_{OLS}$ if they vary across occupations and place. Suppose for simplicity that s_t is constant over time. Then letting $\kappa = \lambda_N + \frac{\beta}{1-\beta}\big(\lambda_L + s_t(\lambda_N - \lambda_L)\big)$, so that $\bar{w}_{jlt}^N = \kappa X_{jt}$, the instrument can be expressed as: $z_{jlt} = s_{jlt} \times \bar{w}_{jlt}^N = s_{jlt} \times \kappa X_{jt}$. Importantly, in this simple example, the instrument z varies across place only due to the national wage-setter share s – the component $\bar{w}$ reflects only aggregate shocks (not indexed by l). The aggregate shocks X_{jt} are orthogonal to Y_{jlt} by definition; if the local wage-setter share s_{jlt} is orthogonal to Y_{jlt}, then our instrument is uncorrelated with the structural residual ϵ, and is therefore a valid instrument for $\bar{w}$ in the model (2).

Estimation

I now return to discussing how I implement this IV design with the Lightcast data. I estimate the regression specification in (2) using a two-stage least squares estimator, instrumenting for $\bar{w}_{jlt}$ with z_{jlt}, using the number of postings for each firm-job-location-month as frequency weights. The instrument z_{jlt} is constructed by interacting national wage-setters' share of postings over the *preceding year*, s_{jlt},[8] with the average wage of national wage-setters in a given occupation-location jl over months $t-1$, $t-2$, and $t-3$. The key identification assumption for this IV estimator to yield a consistent estimate of the population parameter β is an exclusion restriction:

$$E[z_{jlt}\epsilon_{ijlt}] = 0 \tag{4}$$

I interpret the instrument as capturing the (approximate, given logs) contribution of national wage-setters to the local labor market average wage $\bar{w}$, or alternatively

[8] I suppress the dependence of s on past time periods for notational convenience. We can think of this as a 'lagged share' in a shift-share design.

as a measure of predicted exposure to aggregate shocks. Under this interpretation, we can think of the exclusion restriction as an assumption that national wage-setters' relative wage contribution is orthogonal to other determinants of individual firms' posted wages, conditional on fixed effects. I return to these concerns in Section 3.5, where I justify the plausibility of this assumption by ruling out certain plausible confounds.

The IV estimator also requires a standard relevance condition $E[z_{jlt}\bar{w}_{jlt}] \neq 0$. In the simple explicit factor example from Section 3.3, this condition is equivalent to unequal, nonzero factor loadings on aggregate factors X_{jt}; that is, $\lambda_N \neq \lambda_L$. Intuitively, if aggregate factor loadings did not vary between national and local wage-setters, then the share of national wage-setters in a given labor market would be irrelevant for that labor market's exposure to aggregate shocks. In this sense, the strength of this instrument is increasing in the absolute difference $|\lambda_N - \lambda_L|$, and the sign of the first-stage regression coefficient associated with z is determined by the sign of $\lambda_N - \lambda_L$.

The following proposition formalizes the identification assumptions required for the 2SLS estimator to deliver a consistent estimate of β.

Proposition 3 *(Consistency) Consider the structural model (2) and the instrument $z_{jlt} = s_{jlt}\bar{w}^N_{jlt}$. If (i) the relevance condition $E[z_{ljt}\bar{w}_{jlt}|D_{jlt}] \neq 0$ is satisfied, and (ii) the exclusion restriction $E[z_{jlt}\epsilon_{ijlt}] = 0$ holds, then the two-stage least squares estimator of β, $\hat{\beta}_{IV}$ is (root-L) consistent:* $\operatorname{plim}_{L\to\infty} \hat{\beta}_{IV} = \beta$.

This proof is a relatively standard consistency result for just-identified IV with the two-stage least squares estimator, applying the law of large numbers to the relevant sum in the IV sample analog of the IV estimator.

This instrument is a particular type of *exposure design*, characterized by Borusyak, Hull and Jaravel (2022). Under the identification assumptions, the instrument can be interpreted as capturing predicted differential exposure to aggregate (non-place-specific) shocks. This is closely related to the popular shift-share (or Bartik) design (see Goldsmith-Pinkham, Sorkin and Swift (2020)) in the two-industry case. It is also connected to the granular IV design of Gabaix and Koijen (2022), which shares in common an emphasis on distinguishing aggregate from idiosyncratic shocks.

3.4 Estimates

Table 2 presents my baseline IV estimates, estimating the regression specification (2) with two-stage least squares and instrumenting for $\bar{w}_{jlt}$ with z_{jlt}. I cluster standard errors by commuting zone-year; this is intended to be somewhat conservative, and captures the fact that my national wage-setter definitions are based on calendar years. Each column of Table 2 corresponds to a different specification involving different controls and fixed effects. The estimates corresponding to column (1) contain time (month) and occupation-place-year (unit) fixed effects. Columns (2)-(3) report estimates with more-flexible occupation-month fixed effects, and columns (4)-(5) similarly report estimates with place-month fixed effect. Columns (3) and (5) include a control for the log number of postings in local labor market jl over the same three-month period $\bar{w}$ is calculated on. Column (6) my preferred specification, and includes the most flexible set of fixed effects: occupation-time, commuting zone-time, and occupation-commuting zone-firm. In other words, this specification flexibly controls for aggregate time-varying shocks and arbitrary occupation and place-specific shocks, controlling for a broad set of plausible confounds before considering the instrument.

In these specifications, the estimated wage complementarity elasticity ranges between 0.05 and 0.20. My preferred estimates, which correspond to a specification with flexible unit (firm-occupation-location) and time (month-occupation) fixed effects, suggest an elasticity of 0.106, with a standard error of 0.021. Across all the specifications that I consider, the instrument is strong: first stage F-statistics exceed 100 in each specification. Figure 5 presents a binned scatterplot representation of the first-stage regression corresponding to column 2 of Table 1 (with occupation-month fixed effects). This binned scatterplot is a nonparametric visualization of the relationship between the endogenous regressor $\bar{w}$ and the instrument z, after residualizing both $\bar{w}$ and z with respect to the set of fixed effects (in this case, firm-occupation-CZ and occupation-month). The first-stage relationship is strong, monotonic and approximately linear; the first-stage relationship is nearly identical for each of the other five specifications reported in Table 2.

I now turn to characterizing how the estimates vary across observable dimensions of the data. I start by investigating whether these estimates vary systematically across industries. To this end, I consider a variant of my baseline 2SLS specification in which I interact the endogenous regressor $\bar{w}$ and the instrument z with indicator variables corresponding to NAICS supersectors. The results of this exercise are plotted in Figure 6. I use the specification corresponding to column (2) from Table 2, with occupation-time fixed effects. While these estimates are somewhat less precise than my baseline estimates pooling across industries, a few lessons can be gleaned. First, there is suggestive evidence that goods-producing supersectors - construction, manufacturing, and natural resources/mining - exhibit lower wage-posting elasticities. In contrast, the degree of complementarity observed in certain service sectors (information and financial activities in particular) is modestly stronger than my baseline estimates.

Interestingly, my estimates suggest that complementarities in posted wages are stronger among firms that have paid relatively high wages *in the past*. Specifically, I assign each firm-job pair to a *wage quintile*, indicating the quintile associated with postings by that firm in the prior year $y(t) - 1$ relative to other wages for other firms in the same occupation and commuting zone. I estimate my preferred IV specification (e.g. column 6 of Table 2), interacting the endogenous regressor $\bar{w}$ and the instrument z with indicator variables for each quintile, allowing my estimates to flexibly semiparametrically across these bins. The estimated coefficients from this exercise are displayed in Figure 7. This figure document a striking degree of heterogeneity across quintiles of the prior-year wage distribution. Firms that have paid relatively low wages in the prior year exhibit relatively weak wage complementarity, and we cannot reject the null of strategic independence. In contrast, firms that were relatively high-paying over the prior year exhibit very strong responses: for the top 20% of firms, the elasticity of posted wages exceeds 0.32. Notably, this finding is qualitatively consistent with Burdett and Mortensen (1998)-type labor markets; Fukui (2022) shows that the strategic complementarity in wages in this class of models is stronger for more productive, higher-paying firms.

I also investigate whether my estimates are correlated with variation across local labor markets in market concentration. Although I do not observe employment concentration by firms, I can ask whether wage complementarities are correlated with posting concentration. Intuitively, the idea behind this exercise is to ask whether posted wages in local labor markets (job-location pairs) where postings are concentrated in relatively few firms exhibit more or less complementarity than posted wages in local labor markets where postings are more evenly distributed among many firms. To this end, I construct a simple Herfindahl-Hirschman-type index that measures the relative concentration of job postings by employers within each local labor market (occupation-commuting zone) by summing the squared job posting share of each firm i in a given labor market jl in year t. Next, I assign each observation in my panel to the job posting concentration index associated with the posting's occupation, location, and year. Finally, I estimate a variant of my baseline IV specification in which I interact the endogenous regressor and the instrument with a set of indicators for concentration index quintiles. My concentration quintile-specific estimates are depicted in Figure 9. While these estimates are somewhat imprecise, the wage complementarity elasticity is bounded between 0 and 0.20 for each quantile, and there is evidently no systematic pattern across quintiles. In short, there is no substantial evidence that concentration in job postings influences the degree of complementarity in wages. I stop short of interpreting this as evidence against the relevance of local labor market concentration: job postings are an admittedly imperfect proxy for labor market concentration in some environments, and the relevant measure of labor market concentration in this exercise is inherently dependent on an underlying structural model.

Next, I consider extending my baseline design to allow for posted wages to exhibit staggered adjustment to the wages of competitors. My baseline empirical specification carries the implicit assumption that posted wages respond only to competitors' posted wages over the preceding three months - the window over which $\bar{w}_{jlt}$ is computed. If wages adjust slowly, this reduced-form statistical model may be dynamically misspecified, in the sense that the coefficient being estimated does not capture the full response of posted wages to a shock in the mean wage of competitors. To address this important concern, I consider the following dynamic regression specification:

$$w_{ijlt} = \lambda D_{ijlt} + \beta_k \sum_{k \in K} \bar{w}_{jl,t-k} + u_{ijlt} \tag{5}$$

where K indexes a set of lags applied to the endogenous variable $\bar{w}$ (and includes the contemporaneous term $k = 0$). The coefficients $\{\beta_k\}$ capture lagged adjustment. If posted wages respond to competitors' wages immediately, the coefficients on lagged wages will be zero: $\beta_k = 0$ for all $k \neq 0$. This specification includes $|K|$ endogenous regressors; I define $|K|$ instruments in an analogous manner to before, fixing the national wage-setting shares at $t - \max(K)$:

$$\tilde{z}_{jl,t-k} = s_{jl,t-\bar{k}} \times \bar{w}^N_{jl,t-k} \tag{6}$$

where $\bar{k}$ denotes the largest element of K, so that $\tilde{z}$ is defined using the national wage-setter share in the earliest period $t - k$ appearing in the dynamic regression specification. Intuitively, these instruments correspond to each local labor market's predicted exposure in month $t - k$ to aggregate shocks.

I estimate this dynamic specification with two-stage least squares, instrumenting for $\{\bar{w}_{jl,(t-k)}\}_k$ with the set of instruments $\{\tilde{z}_{jl,t-k}\}_k$. It is important to note that this specification explicitly controls for serial correlation in the endogenous regressors $\bar{w}$ and the instruments $\tilde{z}$. Consistency of the 2SLS estimates $\{\hat{\beta}_k\}$ requires exclusion restrictions of the form $E[\tilde{z}_{jl,t-k} u_{ijlt}] = 0$ for all $k \in K$. My estimates for this dynamic specification are described in Table 7. Under the null hypothesis of immediate adjustment in posted wages, the coefficients ought to all be nonzero. In this case, the sum of lagged coefficients is modestly smaller, 0.041 (se 0.067), but a 95% confidence interval includes my baseline estimate of 0.116. In contrast, the sum of lagged coefficients is -0.075 (se 0.061), and we cannot reject a null hypothesis that the sum of lagged coefficients is zero.

As an alternative to this dynamic specification, I also consider a slight generalization of my baseline (static) specification that implicitly accommodates a greater degree of lagged adjustment in the response of posted wages to competitors' wages. Table 9 reports my estimates when the log mean wage of competitors $\bar{w}_{jlt}$ (and the corresponding log mean wage of national wage-setters inside z_{jlt}) are computed based on longer (4 and 6 month) windows. These specifications are nearly identical to my baseline specification: 4 and 6 month windows yield estimates of 0.092 (se 0.025) and 0.110 (se 0.038), respectively, nearly identical to the 0.106 elasticity I obtain with three-month windows in my baseline. Taken together, these results support the view that *posted* wages respond to competitors' posted wages relatively quickly, and so my baseline specification is not missing out on lagged responses.

Next, I turn to some sensitivity checks to investigate the robustness of my empirical results to alternative plausible modeling assumptions. I start by exploring whether my results are sensitive to my definition of local labor markets. My empirical specification has implicitly assumed that labor markets segmented by place-occupation, in the sense that competitors' wages are defined *in the same occupation and commuting zone*. Since we do not directly observe a firm's set of competitors, mis-specification is an important concern: the local labor market defined by occupation-commuting zone pairs may be "too large" or "too small". To concretely illustrate this concern, suppose that firms respond only to competitors in the same *county*, or alternatively that firms respond to wages set by firms for a broader set of occupations. In both cases, the reduced-form model would be mis-specified, and the reduced-form estimand no longer generally corresponds to the (structural) population parameter of interest. In related work examining the influence of employer concentration on wages, Schubert, Stansbury and Taska (2022) engage with a similar challenge in considering the appropriate way to model employer concentration within and across occupations. I address this important concern by considering how my results behave under a variety of different occupational and geographic definitions.

Table 4 reports my baseline estimates under alternative occupational classification systems. The specifications in this table match column 2 of Table 2 by including both firm-job-location and occupation-month fixed effects. Relative to my baseline of SOC 5-digit occupations, I also consider SOC 3-digit and 6-digit classifications and O*NET occupational classifications. Across all of these alternative specifications, the point estimate corresponding elasticity of a firm's posted wage with respect to the mean wage of their competitors ranges from 0.10 to 0.13. This is fairly remarkable, as these alternative occupational classifications vary widely: the three-digit SOC classification has fewer than 100 broad occupation identifiers, whereas the O*NET detailed occupation level has 998. The sample size declines for finer occupational definitions due to sparsity in the panel: as our definition of local labor markets (occupation-place) becomes more narrow, there are more observations in the sample for which we do not observe competitors' wages or national wage-setters.

I also consider how my baseline estimates respond to alternative definitions of geography. Table 5 reports IV estimates in alternative panels defined at the county and state level. The sample is considerably smaller at the county level because $\bar{w}$ is more sparsely populated at more disaggregated spatial units (and vice versa for the state-level estimates in column 3). Reassuringly, estimates with these alternative geographies are broadly similar to the commuting zone-level baseline, with posted wage elasticities between 0.067 and 0.120.

Next, I investigate whether my responses are stronger for larger firms. Unfortunately, I do not observe a relevant measure of firm size (for instance, revenue or employment) in the data. I proxy for firm size by looking instead at the number of job postings for each firm. I associate each job posting observation in the panel with the total number of postings by that firm, and transform this into a quintile of the underlying distribution of postings. I allow my estimates to flexibly vary semi-parametrically with firm postings by interacting the endogenous regressor $\bar{w}$ and the instrument z with a set of posting count quintile indicators. Figure 8 displays the results of this exercise. Although these estimates are somewhat imprecise, the estimated elasticity does not systematically vary across quintiles: firms that post many job postings exhibit approximately the same responses as firms that post relatively few.

I also investigate whether my results vary under sample weights that are constructed to ensure the Lightcast sample of job postings matches the distribution of job openings across industries and states. As described in Section 3.1, while the Lightcast data features nearly universal coverage of online job postings, it is not representative of all job postings in the U.S. Some sectors (e.g. manufacturing) are under-represented relative to their share of job openings in the Job Openings and Labor Turnover Survey (JOLTS), while others (like financial services and information) are over-represented. Likewise, large urban states are over-represented in the Lightcast data. To investigate whether these dimensions are important for my results, I produce sample weights that are constructed to mimic the distribution of postings 2-digit NAICS sectors and states in JOLTS over the period 2010-2023.

Table 6 reports re-weighted specifications under these alternative sample weights. Column (1) reports my baseline (column 2 of Table 1), while columns (2) and (3) report alternative results with sector and state sample weights, respectively. Sector weights yield modestly lower point estimates (0.065; standard error 0.021) relative to my baseline (0.102; standard error 0.020). Intuitively, this reflects higher weight being assigned to goods-producing sector that are under-represented in the Lightcast data and exhibit lower degrees of strategic complementarity, as reported in Figure 6. State weights yield modestly higher estimates (0.163; standard error 0.025). The estimated degree of complementarity in this specification is modestly higher than my baseline, 0.163. Taken together, these results continue to imply a modest and significant degree of strategic complementarity in my sample.

3.5 Threats to Identification

Next, I address threats to identification in this empirical design. Recall that the key identification assumption in this design is that the share of national wage-setters s is orthogonal to other determinants of posted wages (conditional on the control/fixed effect vector D). The objective of this section is to provide evidence that supports the validity of the exclusion restriction.

I start by considering placebo tests with lagged outcomes. The time dimension of the data provides a natural placebo test; if future shocks have an impact on contemporaneous wages, it suggests that the baseline estimates may reflect a spurious relationship between an unobserved shock and the instrument. A "placebo specification" that is estimated with lagged dependent variables therefore provides a powerful and transparent way to rule out some potential violations of the exclusion restriction. Consider the following sequence of models indexed by k:

$$w_{ijl,t+k} = \lambda_k D_{jlt} + \beta_k \bar{w}_{jlt} + \epsilon_{ijl,t+k} \tag{7}$$

which I estimate with 2SLS for $k \in \{-12, -8, -4, 0, 4, 8, 12\}$. I consider 4-month intervals because $\bar{w}$ is defined by aggregating posted wages over the preceding three months prior to t, inducing a mechanical serial correlation in $\bar{w}$ over the preceding three months. I plot these placebo estimates in Figure 10. Reassuringly, the point estimates for specifications involving lagged outcomes are both insignificant and quantitatively much lower in magnitude, with point estimates ranging from -0.03 to 0.04. In contrast, the coefficients on the lead coefficients are relatively larger in magnitude and markedly less precise, consistent with persistent shocks inducing serial correlation in posted wages.

Another important concern for the plausibility of the exclusion restriction concerns our definition of national wage-setters. If local shocks are spatially correlated across commuting zones and national wage-setters tend to operate in relatively few commuting zones in the same region, then local shocks may not "wash out" from the average wages of national wage-setters. For instance, if a firm that employs tractor repair technicians operates in five commuting zones in central Iowa, they may be regarded as a "national wage-setter" according to my baseline definition simply because there is almost no variation in place-specific shocks in the small number of regions in which they operate. I provide two pieces of evidence to address this concern. First, summary statistics suggest that stories like this are relatively rare in practice. Among all national wage-setters (firm-job pairs), the median national wage-setter operates in about 120 commuting zones and 9 states. Only 10% of national wage-setters operate in fewer than 12 commuting zones. That is, under the baseline definition of national wage-setters, the typical national wage-setting firm-occupation operates in a broad swath of states and regions.

To address this concern more concretely, I consider estimating my preferred IV specification with two alternative definitions of national wage-setters that rule out firm-job pairs from being considered national wage-setters if they operate in relatively few CZs. Specifically, I will consider definitions that require firms to operate in (A) 20+ or (B) 50+ commuting zones in each year. For lack of a better term, I will refer to this subset of national wage setters as *geographically diffuse national wage-setters*, or GDNWS. Table 8 describes my preferred specification (with occupation-time and place-time fixed effects) according to these alternative definitions of national wage-setters. In these specifications, GDNWS is good news[9], in the sense that the estimates under these definitions are qualitatively extremely similar to my baseline estimates. Relative to a baseline estimate of 0.106 (se 0.021), estimates under these alternative national wage-setter definitions are 0.092 (se 0.025) and 0.110 (se 0.038) for the cases where national wage-setters are required to operate in 20 or 50 commuting zones, respectively. This robustness check rules out the possibility that my baseline estimates are driven through an 'indirect local shock' channel for national wage-setters that do not operate in many geographic regions.

These checks provide evidence that my results are unlikely to be driven by either place-specific shocks to relatively small wage-setters or persistent unobserved factors that are correlated with lagged outcomes. Moreover, as my preferred specification explicitly accounts for occupation-time and CZ-time fixed effects, we can rule out place-specific, occupation-specific, and aggregate shocks. Taken together, these results greatly constrain the set of possible violations of the exclusion restriction.

Finally, I address an important potential concern, first raised by Batra, Michaud and Mongey (2023), that is relevant for my analysis of online job postings data. Batra, Michaud and Mongey (2023) provide two important caveats to applied work making use of this data. First, the authors stress that relatively few job postings contain wage information. Second, the authors caution that much of the wage information in scraped online job postings post-2017 are *imputed wages/salaries* provided by particular online job boards (a Lightcast 'source'), rather than employers themselves. This second concern is a large potential issue with my design. *A priori*, it is not obvious how the presence of imputed wages leads to bias in my estimates: the direction of bias depends on whether imputed wages are more or less responsive to competing firms' wages than non-imputed wages.

The ideal way to address the concerns induced by imputed wages would be to exclude observations with imputed wages from my analysis. However, the Lightcast data does not directly identify which observations' wages are imputed. I proceed with two methods to address the problem absent this information. First, I consider estimating my preferred IV specification only on sample years 2010-2017, prior to

[9] I have been advised to leave this out of the draft.

major online job boards imputing wages. This ought to exclude the vast majority of imputed wages from the data; however, it is somewhat wasteful, in the sense that it throws out many observations in which wages are not imputed. I also consider a specification in which I exclude sources (online job boards) in which the post-2017 prevalence of nonmissing wage/salary information doubles relative to a 2010-2017 baseline. Table 3 shows my estimates under these two alternative subsamples. My point estimate in the 2010-2017 subsample (column 2) declines modestly to 0.051 (se: 0.035), although a 95% confidence band for these estimates includes my preferred point estimates from the full sample (0.106). Similarly, my point estimate under a sample that excludes all job postings from sources where imputation is a concern (column 3) are 0.121 (se: 0.026). Taken together, these alternative specifications provide some reassurance that imputations from online job boards do not cause a significant amount of bias in my baseline estimates.

3.6 Discussion

It is not obvious how we should interpret these estimates in isolation: is $\beta \approx 0.10$ a significant amount complementarity? This section aims to provide additional context for this question before my structural exploration in Section 4.

One natural way to provide context for these reduced-form estimates is to compare them against estimates of wage-setting complementarities in other environments. Unfortunately, there are relatively few papers that provide empirical evidence on strategic complementarity in wages, or "wage spillovers" across firms more broadly. Derenoncourt et al. (2022) look for evidence of wage changes at big-box retailers in response to voluntary minimum wage increases by a handful of large retailers in the U.S.; the authors do not find evidence of wage changes among competing retailers.[10] In contrast, Staiger, Spetz and Phibbs (2010) find modest evidence of wage increases in the market for nurses following plausibly exogenous wage increases at Veterans Affairs hospitals in the United States. In both of these cases, the analysis was limited to a single industry and a small number of firms/competitors.

Perhaps the best empirical evidence on strategic complementarity in price-setting comes from Amiti, Itskhoki and Konings (2019), who estimate strategic complementarity in goods price-setting using data from the Belgian manufacturing sector. The authors estimate the elasticity of goods prices with respect to the average price of competing goods, the product market analog of the estimand I consider, using an IV design that leverages differential exposure across firms to marginal cost shocks. The

[10]The initial draft of Derenoncourt et al. (2022) reported sizeable wage spillover effects. In a revised draft, the authors show these estimates are spuriously driven by a form of mean reversion in the dependent variable. Revised estimates do not find a significant effect on competing retailers' wages, although these estimates are somewhat less precise.

authors estimate an elasticity of prices with respect to mean competitors' prices of 0.35, indicating a significantly higher degree of complementarity than I find in the context of labor market wage-setting.

Finally, I can consider how my results compare against the degree of strategic complementarity that is implied in labor market specifications commonly encountered in general equilibrium business cycle models. For instance, in monopsonistically (or monopolistically) competitive labor markets with CES labor supply (demand) elasticities, the degree of strategic complementarity is pinned down by the marginal product of labor (marginal rate of substitution between consumption and leisure) and the firm-specific labor supply elasticity. As one concrete example, Mui (2021) develops a monopsonistic New Keynesian model with CES labor supply and decreasing returns to labor. The calibration in this paper ($\alpha = 0.33$, $\epsilon_i = 1$) implies a wage complementarity elasticity of $\beta = 0.25$, modestly higher than my baseline estimates.

In addition, many "medium-scale" dynamic stochastic general equilibrium (DSGE) models incorporate Kimball aggregators over differentiated products/labor varieties precisely to generate strategic complementarity in price and/or wage-setting. For instance, the influential Smets and Wouters (2007) model features monopolistically competitive product and labor markets, where Kimball aggregators over consumption and labor induce variable price and wage markups. This class of models typically assumes a relatively large "Kimball curvature" parameter dictating the hyperelasticity of labor demand, which implies a *very strong* degree of complementarity in wage-setting. For instance, Smets and Wouters (2007) set $\psi = -10$; and Harding, Lindé and Trabandt (2022) set $\psi = -6$. Under flexible wages and unit elastic labor demand, these estimates correspond to $\beta > 0.8$, substantially higher than my empirical estimates.

Taken together, this review suggests that my estimates are modestly weaker than one might expect *a posteriori*, given the empirical evidence on complementarity in goods pricing, or from the implied degree of wage complementarity in commonly-encountered macroeconomic frameworks. In particular, my estimates suggest a dramatically lower degree of wage-setting complementarity than is commonly assumed by medium-scale DSGE models. In the next section of this paper, I address whether this discrepancy is quantitatively relevant through the lens of a particular model.

4 A New Keynesian Model with Wage Complementarity

I now turn to asking whether complementarities in wage-setting are relevant for macroeconomic dynamics. I embed the monopsonistic labor market described in Section 2 into an otherwise-standard New Keynesian model and use this model to investigate how strategic complementarity mediates the propagation of aggregate shocks.

My model closely resembles a textbook New Keynesian model without capital, and collapses to this model as a special case as the firm-specific elasticity of labor supply grows large. Relative to a textbook New Keynesian model with Calvo pricing frictions and Dixit-Stiglitz CES competition in the product market, my model departs in only a few dimensions. First, I assume that labor markets are monopsonistically competitive, with households providing a bundle of differentiated (firm-specific) labor supply to firms. Second, I allow for non-constant equilibrium wage markdowns by assuming that household aggregate labor supply is implicitly defined by a Kimball (1995) aggregator over firm-specific labor varieties. As discussed extensively in Section 2, Kimball labor supply nests the constant-elasticity (CES) benchmark as a special case. In this setup, as in Section 2, wage complementarities can arise through non-constant marginal products or non-constant wage markdowns, and are amplified by the degree of competition in the labor market (as measured by firm-specific labor supply elasticity).

4.1 Model Setup

The economy is populated by a unit mass of identical households and a unit mass of firms indexed by i. Time is discrete and infinite horizon, indexed by $t \in \{0, 1, 2, ...\}$.

Households. There are a large number of identical households. The representative household consumes a bundle of differentiated consumption goods produced by each firm and supplies a bundle of differentiated labor to each firm, $\{c_{it}, \ell_{it}\}$ in each period t. Letting C and L denote aggregate consumption and labor supply, the household maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \frac{L_t^{1+1/\eta}}{1+1/\eta} \right]$$

where $\eta > 0$ denotes the aggregate (Frisch) labor supply elasticity and $\sigma > 0$ denotes the elasticity of intertemporal substitution. Consumption C_t and labor supply L_t are defined by the (CES and Kimball) aggregators:

$$C_t = \Big(\int_0^1 c_{it}^{1-1/\epsilon} di \Big)^{1/(1-1/\epsilon)}$$
$$1 = \int_0^1 \Upsilon \Big(\ell_{it}/L_t \Big) di$$

where, as in Section 2, we require that the function Υ is twice differentiable, strictly convex, and satisfies $\Upsilon(0) = 0$. For simplicity, I assume that the consumption aggregator takes on a "CES" form that implies a constant elasticity of substitution ϵ across varieties.

The household's utility maximization problem is subject to a sequence of period budget constraints and a solvency constraint:

$$\int_0^1 p_{it} c_{it} di + Q_t B_t \leq B_{t-1} + \int_0^1 w_{it} \ell_{it} di + D_t$$
$$\lim_{T \to \infty} E_t \big[Q_T B_T \big] \geq 0$$

where B_t represents holdings in a riskless one-period bond that pays a nominal return of 1 per unit of the bond in period $t+1$, Q_t denotes the price of the bond, w_{it} represents the wage paid per unit of labor supplied to firm i, p_{it} represents the price paid per unit of firm i's consumption good, and D_t represents lump-sum firm profits rebated to the household. The solvency condition is a standard "no-Ponzi" condition ruling out consumption paths that involve endless borrowing (i.e. the household's expected present discounted value of wealth is negative in the long-run), where the bond price Q_T is also the relevant stochastic discount factor in equilibrium.[11]

As in Section 2, I assume a convenient form for the Kimball aggregator function Υ, modified slightly from Dotsey and King (2005):

$$\Upsilon\big(\ell_i/L\big) = \frac{\omega}{1+\psi} \Big[(1+\psi)(\ell_i/L) - \psi \Big]^{1/\omega}$$

where the composite parameter ω is defined as $\omega \equiv \frac{(1+\psi)\phi}{1+\phi\psi}$, with $\phi \in (0,1)$. Recall from Section 2 that ψ is the "Kimball curvature" parameter dictating the departure of firm-specific labor supply from a CES benchmark. I will assume $\psi \leq 0$, corresponding to weakly log-convex firm-specific labor supply.

The representative household's utility maximization problem gives rise to an Euler equation that captures the intertemporal consumption-savings tradeoff and consumption growth:

$$Q_t = \beta E_t \left[\frac{C_t^{1/\sigma}}{C_{t+1}^{1/\sigma}} \frac{P_t}{P_{t+1}} \right] \tag{8}$$

[11]For expediency, I omit discussion of the transversality condition implied by household utility maximization. Given $\sigma > 0$, Walras' law implies the budget constraint binds each period, and the transversality condition implied by household optimality implies that the no-Ponzi condition binds in equilibrium.

Utility maximization gives rise to firm-specific labor supply and consumption demand curves for each variety i:

$$\frac{w_{it}}{W_t} = \left[(1+\psi)(\ell_{it}/L_t) - \psi\right]^{\frac{1-\omega}{\omega}} \tag{9}$$

$$\frac{c_{it}}{C_t} = \left(\frac{p_{it}}{P_t}\right)^{-\epsilon} \tag{10}$$

where p_{it} denotes the price of the consumption good produced by firm i and w_{it} denotes the wage for labor supplied to firm i, respectively, and where the price and wage indices, P_t and W_t, satisfy:

$$P_t = \left(\int_0^1 p_{it}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$$

$$W_t = \frac{1}{\int_0^1 (\ell_{it}/L_t)\left[(1+\psi)(\ell_{it}/L_t) - \psi\right]^{\frac{1-\omega}{\omega}} di}$$

Firms. There is a unit mass of atomistic firms indexed by i. Each firm i produces a differentiated consumption good from labor according to the production technology:

$$Y_{it} = A_t \ell_{it}^{1-\alpha} \tag{11}$$

where A_t is a neutral technology shock and ℓ_{it} represents denotes the quantity of labor hired by firm i in period t from the household.

Each firm i is the sole supplier of distinct variety of the consumption good and the sole purchaser of a distinct variety of labor. Firms maximize profit subject to downward-sloping firm-specific product demand and upward-sloping firm-specific labor supply. Each firm i therefore exercises monopoly power in the product market and monopsony power in the labor market. In addition, I assume that firms nominal rigidities in the form of Calvo (1983) price setting frictions. Relative to a standard New Keynesian model with competitive labor markets, the key difference is that the firm's price-setting problem internalizes the upward-sloping firm-specific labor supply curves they face.

I assume that each firm can costlessly reset their price with probability $1-\theta$ each period. With probability θ, prices remain fixed in nominal terms. The price for firm i's good in period t can therefore be expressed as:

$$p_{it} = \begin{cases} p_{i,t-1} & \text{with probability } \theta \\ p_t^* & \text{with probability } 1-\theta \end{cases} \tag{12}$$

where p_t^* denote the optimal reset price in period t, which is common to all firms that reset prices in a given period t.

The optimal reset price p_t^* maximizes the expected present discounted value of profits for the firm, subject to expected product demand and labor supply. The optimal reset price p_t^* maximizes the expected discounted value of profits that accrue while the reset price p_t^* holds, subject to consumption demand and labor supply:

$$\sum_{k=0}^{\infty} \theta^k E_t \Big[Q_{t,t+k} \Big(p_t^* y_{t+k|t} - \mathcal{C}(y_{t+k|t}) \Big) \Big] \tag{13}$$

where $Q_{t,t+k}$ denotes the stochastic discount factor discounting nominal profits between t and $t+k$, $y_{t+k|t}$ denotes demand in $t+k$ given the last reset price was in period t, and $\mathcal{C}(\cdot)$ denotes the firm's nominal costs of producing a quantity of output $y_{t+k|t}$ in $t+k$ given the price p_t^*. Because this problem is the same for any firm who resets price in period t, I follow convention and suppress the dependence of y, ℓ, and p^* on i.

Importantly, nominal marginal costs $w\ell$ depend on firm-specific labor supply, which implies that the firm's optimal reset price problem accounts for labor market monopsony and strategic interactions in wage-setting through marginal costs. I suppress the dependence of these objects on firm i to emphasize that reset prices are common across all firms in a given period.

The first-order condition for the optimal reset price p_t^* and the derivation of a recursive representation of p_t^* is relatively standard, with the notable exception that the firm accounts for the influence of the reset price on the wage, due to upward-sloping (and potentially non-loglinear) firm-specific labor supply curves. I defer this derivation to the appendix.

Monetary policy and shock processes. I assume that monetary policy, which dictates the nominal interest rate $i_t \equiv -\ln(Q_t)$, follows a simple Taylor rule of the form:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y (Y_t - Y_t^n) + v_t \tag{14}$$

where v_t is a monetary policy shock and Y_t^n denotes natural output, defined as the level of output that would prevail under flexible prices ($\theta = 0$).

I close the model by describing the stochastic processes governing the evolution of aggregate productivity A_t and the monetary policy shock term v_t. I assume that A_t and v_t follow stationary AR(1) processes in logs:

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + \epsilon_t^A \tag{15}$$

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \epsilon_t^v \tag{16}$$

where $\rho_A, \rho_V \in (0,1)$ denote persistence parameters, and the shock terms ϵ^A, ϵ^v are i.i.d. mean zero.

Equilibrium. Equilibrium in this economy is comprised of a sequence of allocations $\{Y_t, C_t, \{c_{it}\}_i, L_t, \{\ell_{it}\}_i, B_t\}_t$ and prices $\{P_t, \{p_{it}\}_i, W_t, \{w_{it}\}_i, Q_t\}_t$ such that, in each time period t: (i) Household optimality: the household's Euler equation, aggregate consumption-labor FOC, firm-specific product demand FOC, and firm-specific labor supply FOC are satisfied, and the household's budget constraint, consumption aggregator, labor supply aggregator, and solvency constraints hold; (ii) Firm optimality: the reset price p_t^* maximizes (13), given household labor supply; (iii) Monetary policy rule: The Taylor rule (14) is satisfied; (iv) Market clearing: product and labor markets clear for each variety i; (v) Shock processes: monetary policy shocks and productivity evolve according to (15) and (16), given initial conditions $\{B_{-1}, A_{-1}, \nu_{-1}\}$.

Existence and uniqueness of equilibrium in this economy are standard. Labor market monopsony and wage complementarity only mediate the slope of the Phillips curve. I log-linearize the model around the zero inflation steady state $\pi_t = P_t/P_{t-1} = 0$, eliminating steady state price and wage dispersion. I defer these relatively standard derivations to Appendix B.2. As a matter of notation, for any endogenous variable x_t, I let $\tilde{x}_t$ denote the log deviation of x from its steady state $\bar{x}$.

The log-linearized New Keynesian Phillips Curve relating inflation to expected inflation and the output gap is $Y_t - Y_t^n$ can be written:

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \kappa\big(\tilde{Y}_t - \tilde{Y}_t^n\big)$$
$$\text{where } \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{\sigma + \frac{1/\eta + \alpha}{1-\alpha}}{1 + \epsilon \frac{(1/\phi - 1)(1-\psi) + \alpha}{1-\alpha}}$$

where the slope of the Phillips curve, κ, is generically decreasing in ϕ and increasing in ψ (decreasing in $-\psi$) for $\psi \leq 0$, $\phi \in (0, 1]$. Y_t^n denotes natural output; that is, the level of output that would prevail in period t under flexible prices. Competitive labor markets correspond to the case where $\phi = 0$ and $\psi = 0$, and monopsonistic competition with CES labor supply corresponds to $\psi = 0$.

As described in Section 2, wage complementarities are implicitly determined by α, ψ, and ϕ. The denominator of the output gap coefficient reflects the impact of non-constant firm-specific labor supply elasticities on inflation dynamics. When $\psi < 0$ (log-convex firm-specific labor supply), this term is positive and decreasing in ψ: thus, a decrease in ψ (greater log-convexity in firm-specific labor supply) flattens the Phillips curve. The intuition behind why wage complementarity flattens the Phillips curve is straightforward. In this model, where complementarities arise from either variable markdowns or variable marginal products, wage complementarities weaken the incentive for firms to change product market prices in response to shocks.

This simple model offers a few important lessons on how wage complementarity matters in a simple New Keynesian environment. To a first-order approximation, wage complementarity mediates macro dynamics by increasing the sensitivity of marginal costs to output; in turn, this reducees a firm's incentive to adjust the price of their good in response to shocks. This is *exactly* the same channel through which monopsony power matters. Second, in this framework, fixing the steady state wage markdown (governed by ϕ) and the output-labor elasticity α, *any* degree of wage complementarity can be realized by choosing the appropriate $\psi < 0$.

4.2 Quantitative Experiments

I now turn to investigating the quantitative relevance of strategic complementarities for the dynamic behavior of this model. The objective of this exercise is to see how the transmission of shocks is mediated by the degree of strategic complementarity in this model, which is implicitly pinned down by the elasticity of output with respect to labor α and the labor supply parameters ψ and ϕ.

Table 10 provides information on my baseline calibration. On the household side, I set the (quarterly) discount rate $\beta = 0.99$, the intertemporal elasticity of substitution $\sigma = 1$ (log utility over aggregate consumption), the elasticity of substitution across product varieties $\epsilon = 9$, and a Frisch elasticity of (aggregate) labor supply of $\eta = 0.2$. On the firm side, I assume the elasticity of output with respect to labor is $1 - \alpha = 0.66$, which intuitively coincides with Cobb-Douglas production with a labor share of 0.66 and fixed (exogenous) capital. I assume that the Calvo parameter $\theta = 0.75$, so that firms reset prices every four quarters. Finally, I choose relatively standard Taylor rule coefficients of 1.5 and 0.125 on inflation and the output gap, respectively.

I consider the behavior of this economy under three different benchmark calibrations of household labor supply. First, I consider a "competitive labor" benchmark with $\psi = \phi = 0$, where as discussed previously the model collapses to a standard three-equation New Keynesian model. Second, I consider a "CES labor" benchmark with $\psi = 0$, $\phi = 0.5$, corresponding to a firm-specific labor supply elasticity of 1. Last, I consider a "Kimball labor" benchmark with $\psi = -6$ and $\phi = 0.5$, intended to roughly approximate the Kimball curvature widely utilized in medium-scale DSGEs with imperfectly competitive labor markets, including Smets and Wouters (2007) and Harding, Lindé and Trabandt (2022).

I consider the impulse responses associated with a persistent monetary policy shock. Specifically, I consider a 25 basis point (100 bp annualized) unanticipated contractionary monetary policy shock that hits the steady state of the economy at $t = 0$. I set the AR(1) autoregressive parameter of the monetary policy shock process to $\rho_v = 0.75$, implying a half-life of just over two quarters. Selected impulse responses to this shock for each of my three benchmark calibrations are depicted in

Figure 11. Both monopsonistic calibrations exhibit substantially larger responses of real aggregates, and substantially weaker responses to inflation, relative to a competitive benchmark. The reason why wage-setting complementarity amplifies the response of real aggregates can be made clear by contrasting the CES mononopsony benchmark with the Kimball monopsony benchmark. On impact, households observe the contractionary monetary policy shock and substitute away from present consumption. Firms that can reset prices in the impact period $t = 0$ would like to lower their price. When labor supply is log-convex, this requires paying a relatively higher wage: marginal costs are relatively steeper in output. This weakens the incentive for firms to lower their product price, so output (and thus employment) fall relatively more in the Kimball calibration.

As a second exercise, I consider the impulse responses of this economy to a persistent productivity shock. Specifically, I consider a 1 unit increase in productivity that hits the steady state of the economy at $t = 0$ with an AR(1) persistence of 0.90 (implying a half-life of about 6.5 quarters). Selected impulse responses for this exercise are depicted in Figure 12. The dynamic impacts of this persistent productivity shock are qualitatively similar to the monetary shock – strategic complementarity through variable markdowns and/or monopsony power are associated with greater movement in real aggregates and less movement in inflation due to the same marginal cost channel as before.

Finally, I consider stochastic simulations of the model. One simple measure of monetary non-neutrality is the variance of real output in the model. I consider stochastic simulations in which fluctuations are driven entirely by monetary shocks I assume that i.i.d. monetary shocks are normally distributed, $\epsilon_\nu \sim N(0, \sigma_\nu^2)$, with $\sigma_\nu = 0.25$, which implies that a one standard deviation monetary shock is 100 basis points annualized. As before, I set the persistence of the monetary shocks to $\rho_\nu = 0.75$. Table 11 shows how the variance of real output and inflation depend on α and the labor supply parameters ϕ, ψ. Panel A presents simulations with $\alpha = 0.33$, and Panel B presents simulations with linear production. As with the impulse response functions, calibrations with greater strategic complementarity exhibit more monetary non-neutrality. A strong degree of curvature in log labor supply, e.g. $\psi = -6$, leads to much larger output fluctuations than either a CES monopsonistic or competitive benchmark. The mapping from the magnitude of wage complementarity to real output and inflation is mediated by the slope of the Phillips curve; all else equal, stronger wage complementarity is associated with a lower κ and a correspondingly flatter Phillips curve. As a result, the relative contribution of wage complementarity to monetary non-neutrality will depend heavily on the slope of the Phillips curve in the absence of complementarities. For instance, in Panel B, with linear production $\alpha = 0$ this model exhibits a relatively steep Phillips curve under the baseline calibration. Comparing the model with competitive labor markets against the model with monopsonistic labor markets and Kimball labor supply with $\psi = -6$, the volatility of real output grows dramatically - by a factor of 14. In contrast, if we compare the same calibrations with $\alpha = 0.33$ (where the initial slope of the Phillips curve is 0.129), the same monopsonistic Kimball specification increases the volatility of labor by a factor of 2.5 relative to the competitive labor markets case.

Taken together, these quantitative experiments show that strategic complementarity in wage-setting is a potentially powerful amplification mechanism for nominal shocks. Wage complementarity is intimately related to monopsony power in this model, and operates through the same channel: greater complementarity in wages increases the sensitivity of firms' marginal costs to output, dampening the responsiveness of price adjustment and inflation, and therefore amplifying the responsiveness of real quantities. In turn, this mechanism is captured by the slope of the Phillips curve: greater wage complementarity flattens the slope of the New Keynesian Phillips Curve.

5 Conclusion

This paper has explored the empirical and theoretical relevance of strategic complementarities in wage-setting. I start by developing a reduced-form empirical design to estimate the elasticity of firms' posted wages with respect to the wages of competing firms. This design leverages plausibly exogenous variation in the density of national wage-setters across local labor markets as a source of differential exposure to aggregate shocks. My estimates suggest that the elasticity of posted wages with respect to the mean posted wage of competing firms is between 0.05 and 0.15.

I provide context for my empirical estimates by considering the implied magnitude of wage-setting complementarity in labor market environments typically found in macroeconomic models. The implied magnitude of strategic interactions in wage-setting varies dramatically in practice, depending both on specific calibrations and on functional form assumptions imposed a priori. For instance, "medium-scale" dynamic stochastic general equilibrium models in the vein of Smets and Wouters (2007) rely on variable wage markups[12] to generate substantial complementarity in wages, with a complementarity elasticity of approximately 0.90. In contrast, in a simple monopsonistic labor market with CES labor demand and a constant marginal product of labor, wages are strategically independent. My empirical estimates suggest that the complementarity elasticity of wages is low, between 0.10 and 0.20, indicating that in practice, we observe relatively modest complementarity.

[12] These models typically assume labor market power resides with households (monopolistically competitive labor markets); when wages are flexible, the optimal wage is a markup over the marginal rate of substitution between consumption and leisure.

In the final part of this paper, I investigate whether the difference in these benchmarks has any quantitative relevance for macroeconomic dynamics, with a special focus on the effect of monetary policy. I develop a New Keynesian model with monopsonistic labor markets, where complementarities can arise due to non-constant wage markdowns and marginal products. This setup nests a competitive labor market and a monopsonistic labor market with constant labor supply elasticities as special cases, and allows me to characterize how wage-setting complementarities impact model dynamics. I demonstrate that strategic complementarity in wage-setting is a potentially powerful source of real rigidity in this model, amplifying output fluctuations in response to nominal shocks and flattening the New Keynesian Phillips Curve that dictates the equilibrium relationship between inflation and output. At the same time, I show that the high degree of complementarity exhibited by the aforementioned quantitative DSGEs likely over-estimates the importance of wage complementarities for macroeconomic dynamics.

This paper has two primary contributions. First, my estimates provide a valuable "portable moment" that can be used to discipline the calibration of models with imperfectly competitive labor markets. This is important because imperfectly competitive labor markets often imply a degree of strategic complementarity through other parameters. To my knowledge, my estimates are the first reduced-form estimates of wage spillovers that are not limited to a particular industry or sector (e.g. nurses or big-box retailers). Moreover, my estimand - the elasticity of (posted) wages with respect to the mean wage of competitors - can be easily interpreted in a broad variety of imperfectly competitive models. My estimates can therefore be used both as a way to discipline and also distinguish between labor market environments and calibrations.

Second, the exposure-based empirical design that I develop has potential uses in other price-setting environments where uniform pricing across locations is observed. For instance, Dellavigna and Gentzkow (2019) document that uniform price-setting across establishments is relatively common among U.S. retailers. A similar exposure-based design could be used in this environment to investigate the degree of strategic complementarity observed in retail prices.

I conclude by suggesting a few directions for future research. First, while my estimates provide evidence on the degree of complementarity observed in *posted* wages, they do not inform us about the degree of complementarity observed in wages for incumbent workers. Follow-up analysis that studies strategic interactions between firms in the wages for incumbent workers would provide a natural complement to the results I present here. In a related vein, my quantitative model is narrowly focused on the class of imperfectly competitive labor markets with atomistic wage-setters labor market clearing, both because this class of labor markets is common in the literature, and because it admits a relatively transparent relationship between model primitives and the degree of wage complementarity. I defer a quantitative analysis of strategic complementarity in, for instance, environments with granular firms or search and matching frictions for future research.

References

Alvarez, Fernando E., Francesco Lippi, and Takis Souganidis. 2022. "Price Setting with Strategic Complementarities as a Mean Field Game." *NBER working paper #30193*.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. 2019. "International Shocks, Variable Markups, and Domestic Prices." *The Review of Economic Studies*, 86(6): 2356–2402.

Autor, David, and David Dorn. 2013. "The Growth of Low Skill Service Jobs and the Polarization of the U.S. Labor Market." *The American Economic Review*, 103(5): 1553–1597.

Ball, Laurence, and David Romer. 1990. "Real Rigidities and the Non-Neutrality of Money." *The Review of Economic Studies*, 57(2): 183–203.

Batra, Honey, Amanda Michaud, and Simon Mongey. 2023. "Online Job Posts Contain Very Little Wage Information." *NBER Working Paper 31984*.

Blanchard, Olivier. 1986. "The Wage Price Spiral." *The Quarterly Journal of Economics*, 101(3): 543–566.

Borusyak, Kirill, Peter Hull, and Xavier Jaravel. 2022. "Quasi-Experimental Shift-Share Research Designs." *The Review of Economic Studies*, 89(1): 181–213.

Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer. 1985. "Multimarket Oligopoly: Strategic Substitutes and Complements." *Journal of Political Economy*, 93(3): 488–511.

Burdett, Kenneth, and Dale T. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review*, 39(2): 257–273.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12: 383–398.

Dellavigna, Stefano, and Matthew Gentzkow. 2019. "Uniform Pricing in U.S. Retail Chains." *The Quarterly Journal of Economics*, 134: 2011–2084.

Derenoncourt, Ellora, Clemens Noelke, David Weil, and Bledi Taska. 2022. "Spillover Effects from Voluntary Employer Minimum Wages." *NBER Working Paper #29425*.

Dotsey, Michael, and Robert G. King. 2005. "Implications of State-dependent Pricing for Dynamic Macroeconomic Mdels." *Journal of Monetary Economics*, 52: 213–242.

Fukui, Masao. 2022. "A Theory of Wage Rigidity and Unemployment Fluctuations with On-the-Job Search." *Working paper.*

Gabaix, Xavier, and Ralph Koijen. 2022. "Granular Instrumental Variables." *NBER Working Paper #28204.*

Gertler, Mark, and Antonella Trigari. 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *The Journal of Political Economy*, 117(1).

Goldsmith-Pinkham, Paul, Isaac Sorkin, and Henry Swift. 2020. "Bartik Instruments: What, When, Why, and How." *American Economic Review*, 110(8): 2586–2624.

Golosov, Mikhail, and Jr. Lucas, Robert E. 2007. "Menu Costs and Phillips Curves." *Journal of Political Economy*, 115: 171–199.

Harding, Martín, Jesper Lindé, and Mathias Trabandt. 2022. "Resolving the Missing Deflation Puzzle." *Journal of Monetary Economics*, 126: 15–34.

Hazell, Jonathan, Christina Patterson, Heather Sarsons, and Bledi Taska. 2022. "National Wage Setting." *NBER Working Paper #30623.*

Kimball, Miles S. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit and Banking*, 27(4): 1241–1277.

Mui, Preston. 2021. "Labor Market Monopsony in the New Keynesian Model: Theory and Evidence." *Working paper.*

Nakamura, Emi, and Jón Steinsson. 2010. "Monetary Non-neutrality in a Multi-sector Menu Cost Model." *The Quarterly Journal of Economics*, 125(3): 961–1013.

Nakamura, Emi, and Jón Steinsson. 2018. "Identification in Macroeconomics." *Journal of Economic Perspectives*, 32(3): 56–86.

Schubert, Gregor, Anna Stansbury, and Bledi Taska. 2022. "Employer Concentration and Outside Options." *Working paper.*

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97: 586–606.

Staiger, Douglas, Joanne Spetz, and Ciaran S. Phibbs. 2010. "Is There Monopsony in the Labor Market? Evidence from a Natural Experiment." *Journal of Labor Economics*, 28(2): 211–236.

Figures and Tables

Table 1: Summary Statistics for Job Postings and Panel Data

	Mean	Std. Dev.	p25	p75	Obs.
A. Baseline Sample					
Posted wage (2020 $, thousands)	60.6	40.9	33.8	75.0	19,278,446
Competitor posting count	1342	1986	267	1559	19,278,446
Indicator: Full-time	0.804	0.397	1	1	19,278,446
Indicator: Min. bachelor's degree	0.329	0.470	0	1	8,165,096
B. National Wage Setters					
Posted wage (2020 $, thousands)	77.1	56.3	44.8	91.0	776,791
Competitor posting count	609.7	1204	46	619	776,791
Indicator: Full-time	0.800	0.400	1	1	776,791
Indicator: Min. bachelor's degree	0.378	0.485	0	1	229,693
C. Local Wage Setters					
Posted wage (2020 $, thousands)	59.9	40.0	33.5	73.5	18,501,655
Competitor posting count	1372	2000	284	1589	18,501,655
Indicator: Full-time	0.804	0.397	1	1	18,501,655
Indicator: Min. bachelor's degree	0.327	0.469	0	1	7,934,403

Notes: This table provides summary statistics for the baseline panel that is an input into my empirical estimation. Panel A documents summary statistics for all postings; Panels B and C document postings for national wage-setters and local (non-national) wage-setters, respectively. See Section 3.1 for more details.

Table 2: Baseline IV Estimates

Outcome: w_{ijlt}	(1)	(2)	(3)	(4)	(5)	(6)
$\bar{w}_{ijlt}$	0.212	0.102	0.102	0.206	0.219	0.106
	(0.021)	(0.020)	(0.019)	(0.024)	(0.007)	(0.021)
$\ln(v_{jlt})$			0.000		0.007	
			(0.001)		(0.002)	
Fixed effects:						
Firm-Occ-CZ	x	x	x	x	x	x
Time	x					
Time-Occ		x	x			x
Time-CZ				x	x	x
1^{st} stage F-stat.	540	486	495	435	457	418
Observations	16,155,759	16,154,519	16,154,519	16,152,701	16,152,701	16,151,512

Notes: This table presents my baseline 2SLS estimates of (2), using z as an instrument for $\bar{w}$. Each column corresponds to a different set of fixed effects and controls. Standard errors are clustered by CZ-year. See Section 3.4 for more details.

Table 3: IV Estimates, Excluding Possible Imputed Wages

	Baseline	Alternative Samples	
		2010-2017	Excluding Likely Imputed
Outcome: w_{ijlt}	(1)	(2)	(3)
$\bar{w}_{jlt}$	0.106	0.051	0.121
	(0.021)	(0.035)	(0.026)
1[st] stage F-statistic	540	101	339
Observations	16,151,512	1,922,790	7,999,648

Notes: This table presents estimates for baseline IV regression specification of equation (2) under alternative samples. Column (1) reports my preferred estimates from Table 2. All reported specifications incorporate firm-occupation-location and occupation-month fixed effects. Standard errors are clustered by CZ-year. See Section 3.5 for more details.

Table 4: IV Estimates, Alternative Occupational Classifications

	Baseline	Alternative Definitions		
	SOC-5	SOC-3	SOC-6	O*NET
Outcome: w_{ijlt}	(1)	(2)	(3)	(4)
$\bar{w}_{jlt}$	0.106	0.104	0.131	0.133
	(0.021)	(0.020)	(0.023)	(0.026)
1^{st} stage F-statistic	540	483	397	404
Occupations	447	97	790	998
Observations	16,151,512	27,264,885	13,005,091	12,460,396

Notes: This table presents alternative estimates of the baseline IV regression specification of equation (2) with alternative occupational definitions. All reported specifications incorporate firm-occupation-location and occupation-month fixed effects. Standard errors are clustered by CZ-year. See Section 3.4 for more details.

Table 5: IV Estimates, Alternative Geographic Units

	Baseline	Alternative Definitions	
	Commuting Zone	County	State
Outcome: w_{ijlt}	(1)	(2)	(3)
$\bar{w}_{jlt}$	0.102	0.067	0.124
	(0.020)	(0.021)	(0.032)
1^{st} stage F-statistic	540	539	235
Geographic Units	740	1190	50
Observations	16,151,512	4,659,512	27,073,507

Notes: This table presents alternative estimates of the baseline IV regression specification of equation (2) with alternative geographic definitions. All reported specifications incorporate firm-job-location and occupation-month fixed effects. Standard errors are clustered by CZ-year. See Section 3.4 for more details.

Table 6: IV Estimates, JOLTS Sample Weights

	Baseline	Sample Weights	
		Sector	State
Outcome: w_{ijlt}	(1)	(2)	(3)
$\bar{w}_{jlt}$	0.106	.064	0.161
	(0.021)	(.022)	(0.025)
1^{st} stage F-statistic	418	450	318
Observations	16,151,512	16,151,512	16,151,512

Notes: This table presents alternative estimates of the baseline IV regression specification of equation (2) with sample weights constructed to match the distribution of job postings across sectors (column 2) and states (column 3), as reported by JOLTS. See Section 3.4 for more details.

Table 7: IV Estimates, Dynamic Specification

Outcome: w_{ijlt}	Coefficient (1)	Standard error (2)
$\bar{w}_{jlt}$	0.116	0.052
$\bar{w}_{ijl,(t-3)}$	-0.075	0.047
$\bar{w}_{ijl,(t-6)}$	-0.039	0.047
$\bar{w}_{ijl,(t-9)}$	0.039	0.053
Sum of coefficients	0.041	0.067
Sum of lagged coefficients	-0.075	0.061
Observations	9,655,218	

Notes: This table presents IV estimates for the dynamic specification with lagged dependent variables described in Section 3.4. Column 1 reports estimated coefficients and Column 2 reports standard errors. The first row corresponds to the estimated loading on competitors' mean posted wage over the preceding 3 months $t-1, t-2, t-3$. The next three rows correspond to the estimated loadings on lags of the competitors' mean posted wage. This regression specification includes firm-occupation-location, occupation-time, and location-time fixed effects. Standard errors are clustered by CZ-year. See Section 3.4 for more details.

Table 8: IV Estimates, Alternative National Wage-Setter Defs.

	Baseline	Alternative NWS Defs.	
	$\geq$ 3 CZs	$\geq$ 20 CZs	$\geq$ 50 CZs
Outcome: w_{ijlt}	(1)	(2)	(3)
$\bar{w}_{jlt}$	0.106	0.092	0.110
	(0.021)	(0.025)	(0.038)
1^{st} stage F-statistic	418	299	203
Observations	16,151,512	10,843,082	8,097,991

Notes: This table presents IV estimates under alternative definitions of national wage-setters. Column 1 reoprts my preferred estimates, Column 6 of Table 2. Column 2 reports estimates when national wage-setters are required to operate in at least 20 commuting zones in a calendar year. Column 3 reports estimates when national wage-setters are required to operate in at least 50 commuting zones in a calendar year. All specifications include firm-occupation-location, occupation-time, and location-time fixed effects. Standard errors are clustered by CZ-year. See Section 3.4 for more details.

Table 9: IV Estimates, Alternative Competitor Wage Windows

	Baseline	Alternative Specifications	
	3-month	4-month	6-month
Outcome: w_{ijlt}	(1)	(2)	(3)
$\bar{w}_{jlt}$	0.106	.097	0.084
	(0.021)	(.021)	(0.023)
1^{st} stage F-statistic	418	499	439
Observations	16,151,512	16,584,820	17,033,394

Notes: This table presents alternative estimates of the baseline IV regression specification with alternative lag windows for constructing the mean wage of competitors $\bar{w}_{jlt}$ and the instrument z_{jlt}. Column 1 reports my baseline preferred specification, which calculates $\bar{w}_{jlt}$ with a three-month window ($t-1$, $t-2$, $t-3$). Columns 2 and 3 report estimates where $\bar{w}$ is constructed with 4 and 6-month windows, respectively. All specifications include firm-occupation-location, occupation-time, and location-time fixed effects. Standard errors are clustered by CZ-year. See Section 3.4 for more details.

Table 10: Calibrated Model Parameters

Parameter	Description	Value
β	Discount factor (quarterly)	$0.99^{1/4}$
σ	Intertemporal elasticity of substitution	1
ϵ	Product demand elasticity	9
η	Frisch (aggregate) labor supply elasticity	0.2
ϕ	Kimball: Gross wage markdown	1 or 0.5
ψ	Kimball: log labor supply curvature	0 or 1
$1-\alpha$	Elasticity of output with respect to labor	0.66
θ	Calvo price stickiness	0.75
ϕ_π	Taylor rule: output gap coefficient	0.125
ϕ_π	Taylor rule: inflation coefficient	1.5

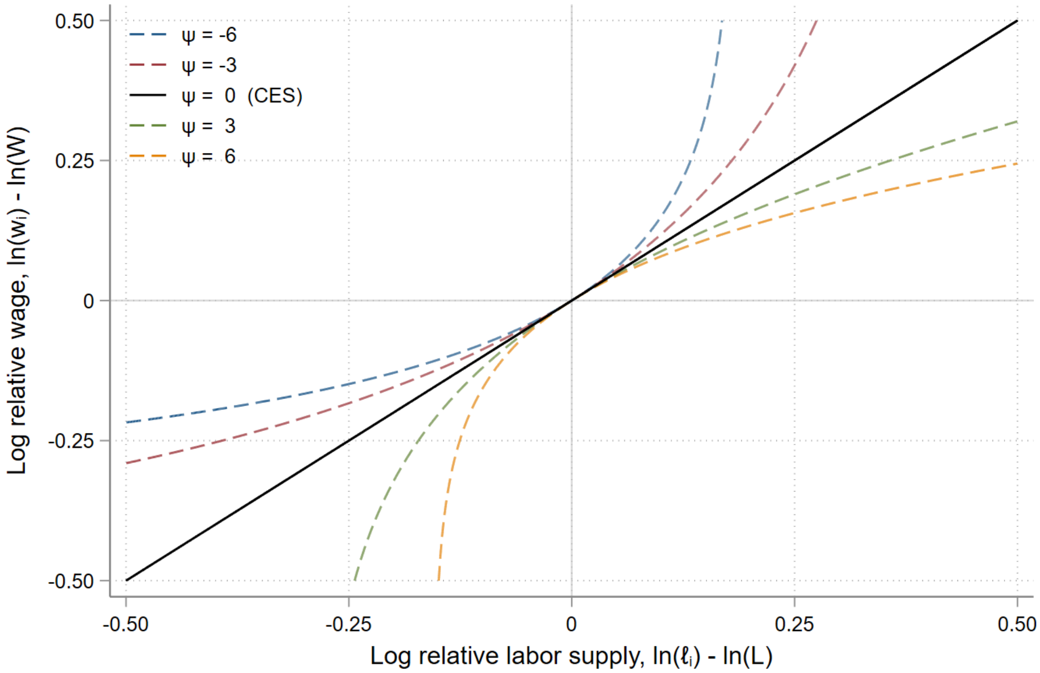
Notes: This table describes the baseline calibration of the model in 4. See Section 4 for more details.

Table 11: Monetary Non-Neutrality and Wage Complementarity

Labor Supply Calibration			PC slope	Simulated Moments	
Description	ϕ	ψ	κ	$\text{Var}(\hat{Y}_t)$	$\text{Var}(\hat{\pi}_t)$
A. $\alpha = 0.33$					
Competitive	1	0	0.129	0.259	1.036
Monopsonistic, CES	0.5	0	0.061	0.479	0.424
Monopsonistic, Kimball	0.5	-6	0.034	0.650	0.179
B. $\alpha = 0$					
Competitive	1	0	0.515	0.041	2.651
Monopsonistic, CES	0.5	0	0.094	0.347	0.734
Monopsonistic, Kimball	0.5	-6	0.042	0.589	0.251

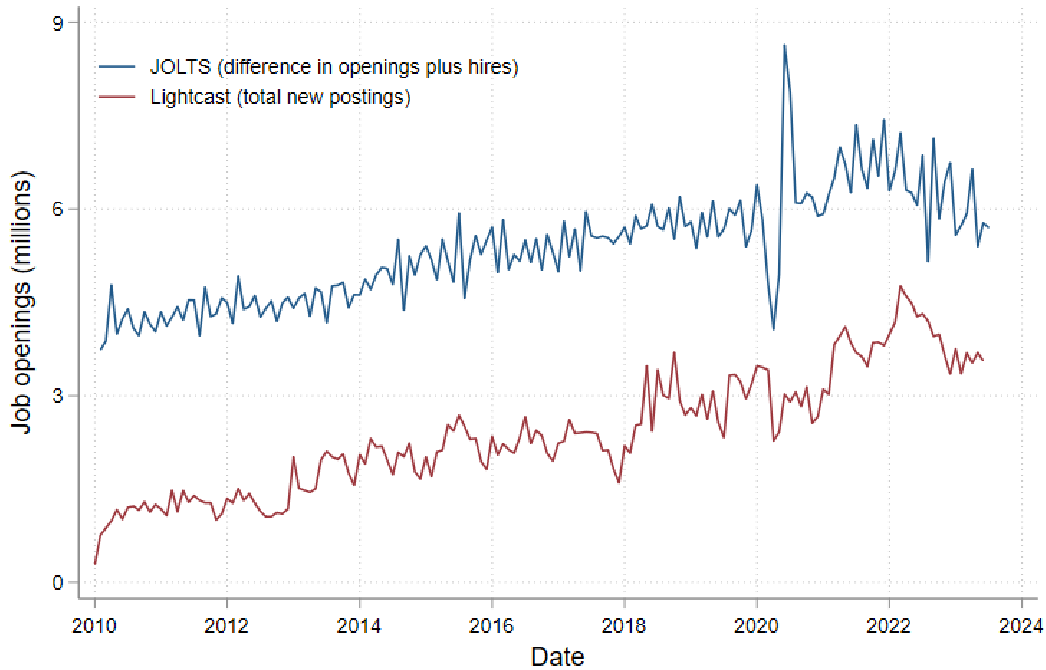
Notes: This table describes the results from stochastic simulations of the model. All parameters except α, ψ, and ϕ are given in Table 10. Each row in this table corresponds to a different calibration of α, ψ, and ϕ. Panel A corresponds to calibrations with $\alpha = 0.33$ (output-labor elasticity of 0.67). Panel B corresponds to calibrations with $\alpha = 0$ (linear production). See Section 4 for more details.

Figure 1: Kimball Labor Supply System



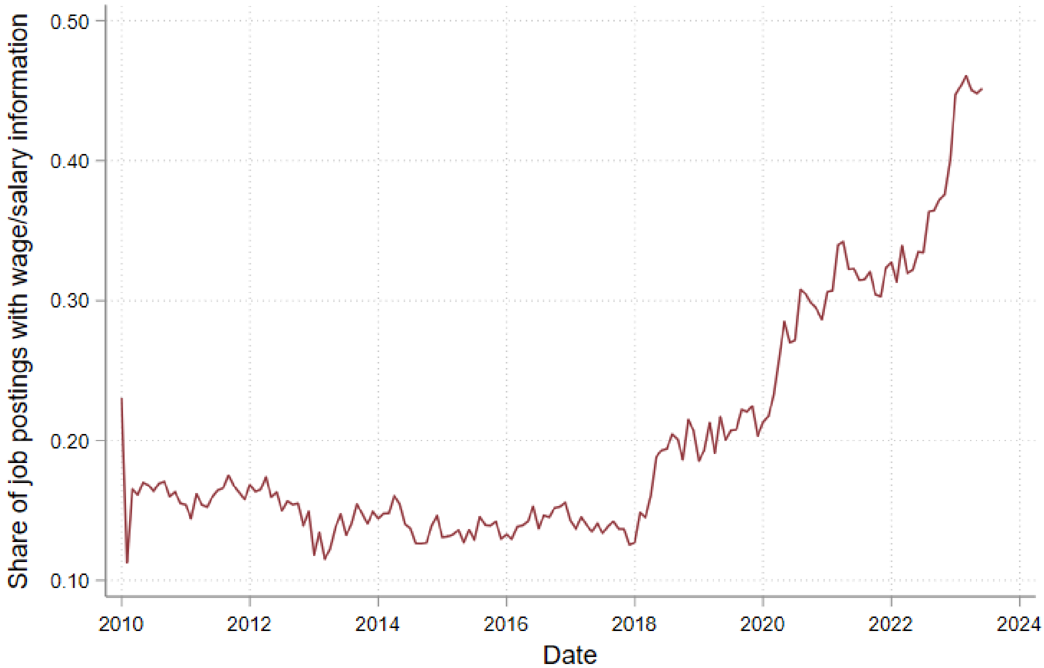
Notes: This figure documents the Kimball labor supply system used in the Section 2 model. Specifically, this figure plots firm-specific (relative) labor supply and relative wages under several parameterizations of the Kimball aggregator Υ. I set the parameter $\phi = 0.5$, corresponding to a unit elasticity of labor supply when $w_i = W$. The parameter ψ determines the degree of departure from a constant firm-specific labor supply elasticity benchmark. See Section 2 for more details.

Figure 2: Job Openings in JOLTS vs. Lightcast



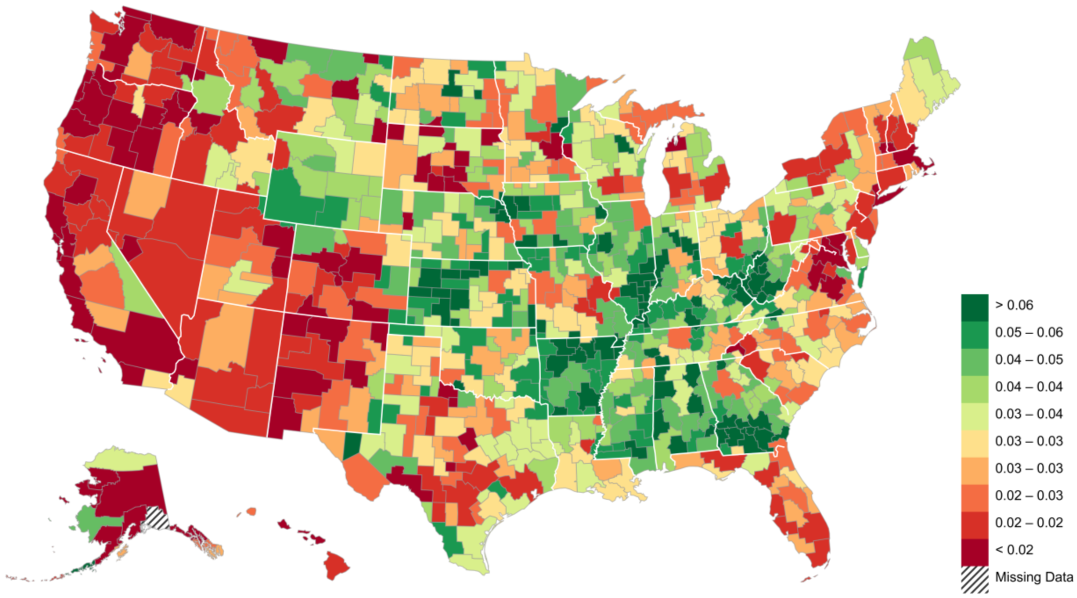
Notes: This figure compares job openings from the Bureau of Labor Statistics' Job Openings and Labor Turnover Survey (JOLTS) against Lightcast. The JOLTS series corresponds to the difference in job openings between months t and $t-1$ (where job openings in t measure the number of unfilled job openings on the last day of the month) plus the number of hires in month t. The Lightcast series corresponds to the sum of job openings posted in month t. See Section 3.1 for more details.

Figure 3: Lightcast Job Postings with Wage/Salary Information



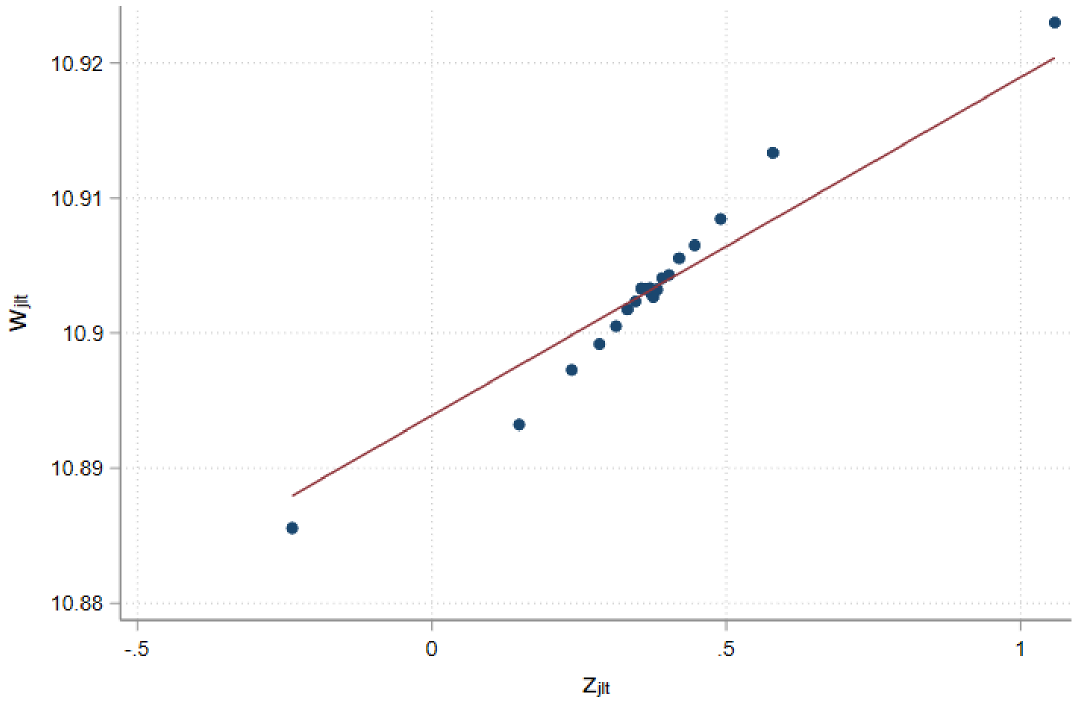
Notes: This figure documents the share of job postings with wage/salary information in the Lightcast posting-level data.

Figure 4: National Wage-Setter Shares Across Commuting Zones



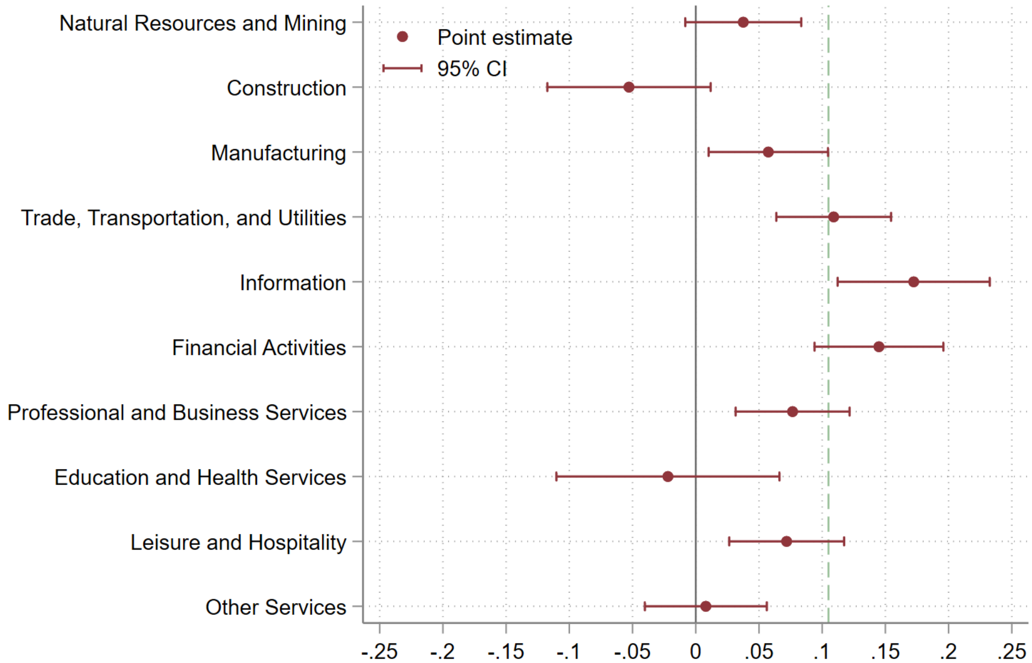
Notes: This figure depicts the share of postings due to firm-occupation pairs that are categorized as national wage-setters in the baseline sample across commuting zones in my full analysis sample (Jan. 2010 - Jul. 2023). Individual commuting zones are colored based on their decile in the (unweighted) distribution of national wage-setter shares across commuting zones. See Section 3.2 for more details.

Figure 5: IV Estimates: First Stage Binned Scatterplot



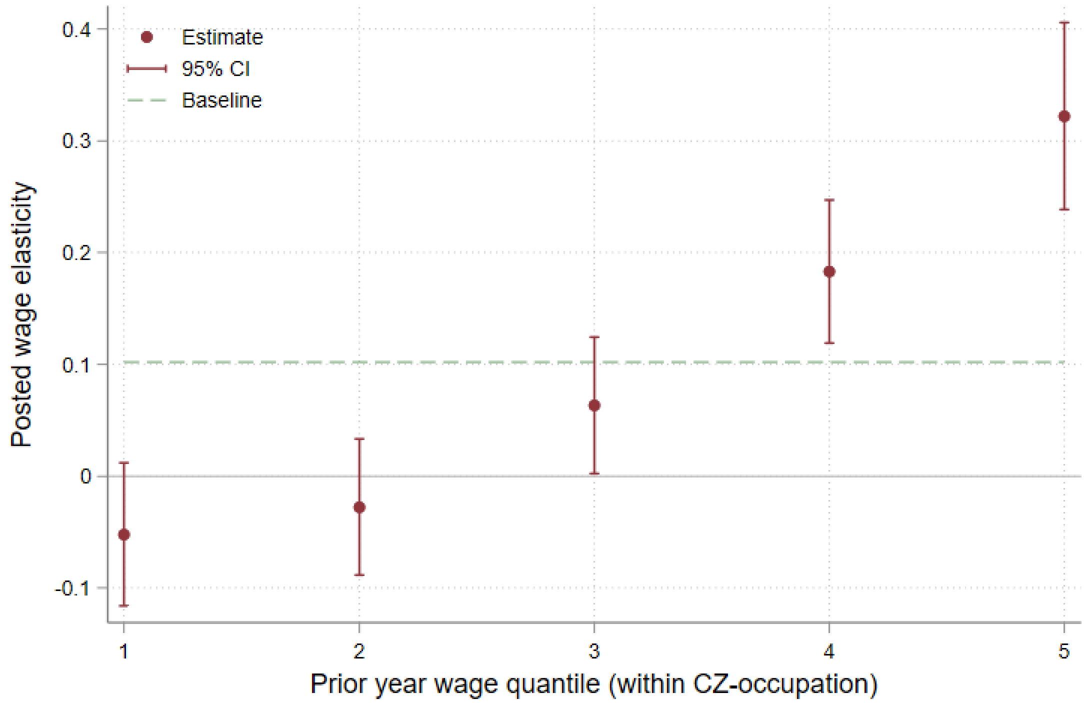
Notes: This figure presents a binned scatterplot of the relationship between the endogeneous regressor $\bar{w}$ and the instrument z, absorbing firm-occupation-CZ and occupation-month fixed effects. See Section 3.4 for more details.

Figure 6: IV Estimates: Heterogeneity by NAICS Industry



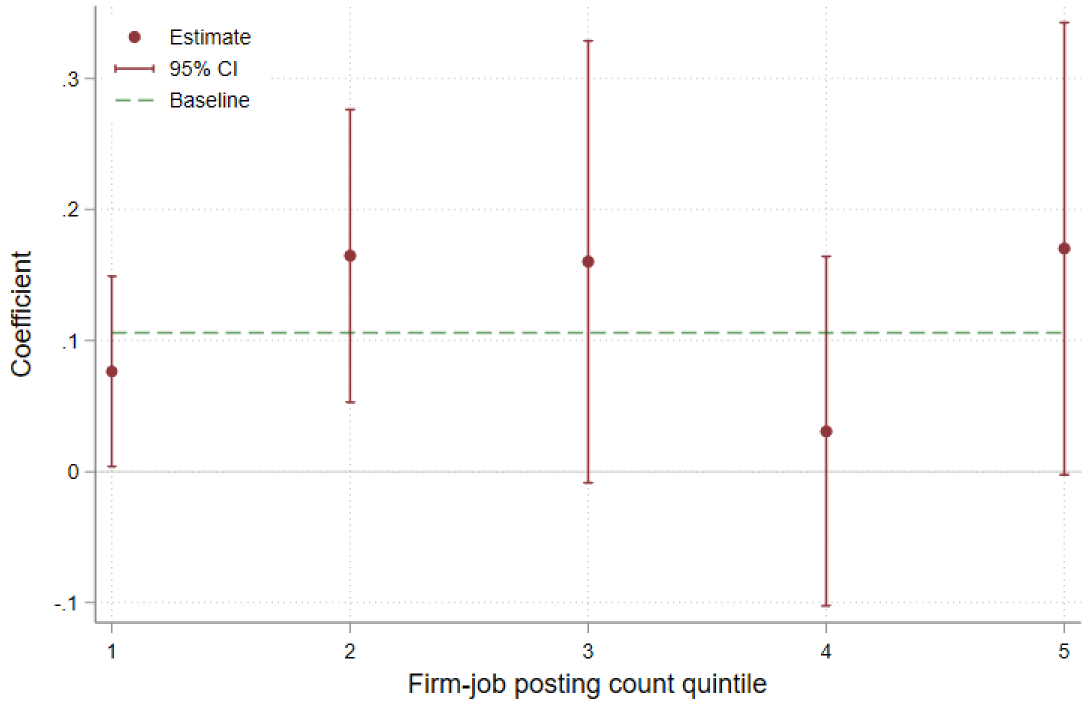
Notes: This figure documents heterogeneity in the baseline empirical design across NAICS supersectors. See Section 3.4 for more details.

Figure 7: IV Estimates: Heterogeneity by Prior Year Wage Quantile



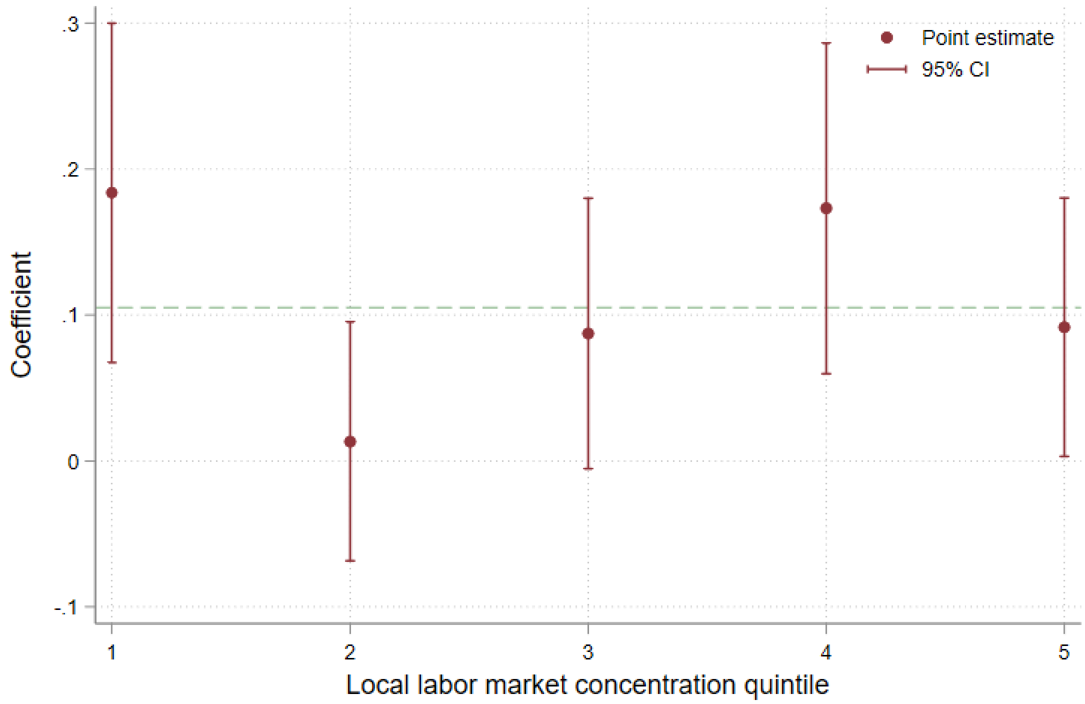
Notes: This figure documents heterogeneity in the baseline empirical design across *prior year wage quantiles*. Firm-occupation-place observations are partitioned into quantiles based on their position in the distribution of wages within occupation-place in the prior calendar year. The instrument and endogenous regressor are interacted with indicator variables for each quantile. See Section 3.4 for more details.

Figure 8: IV Estimates: Heterogeneity by Firm Posting Counts



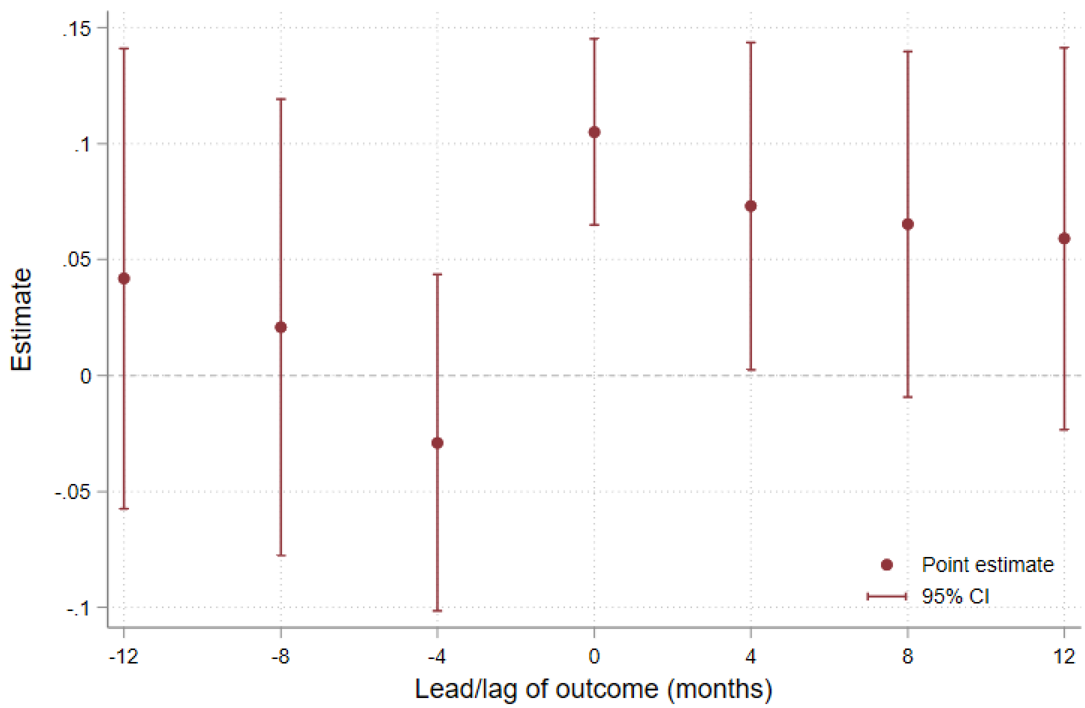
Notes: This figure documents heterogeneity in my baseline estimates by posting count, as a rough proxy for firm size. I tabulate the total number of postings for each firm-job pair, and partition each observation in my panel into quintiles based on their position in the distribution of overall postings across firm-job pairs. I then interact the endogenous regressor $\bar{w}$ and the instrument z with indicators for each quintile.

Figure 9: IV Estimates: Heterogeneity by Local Labor Market Concentration



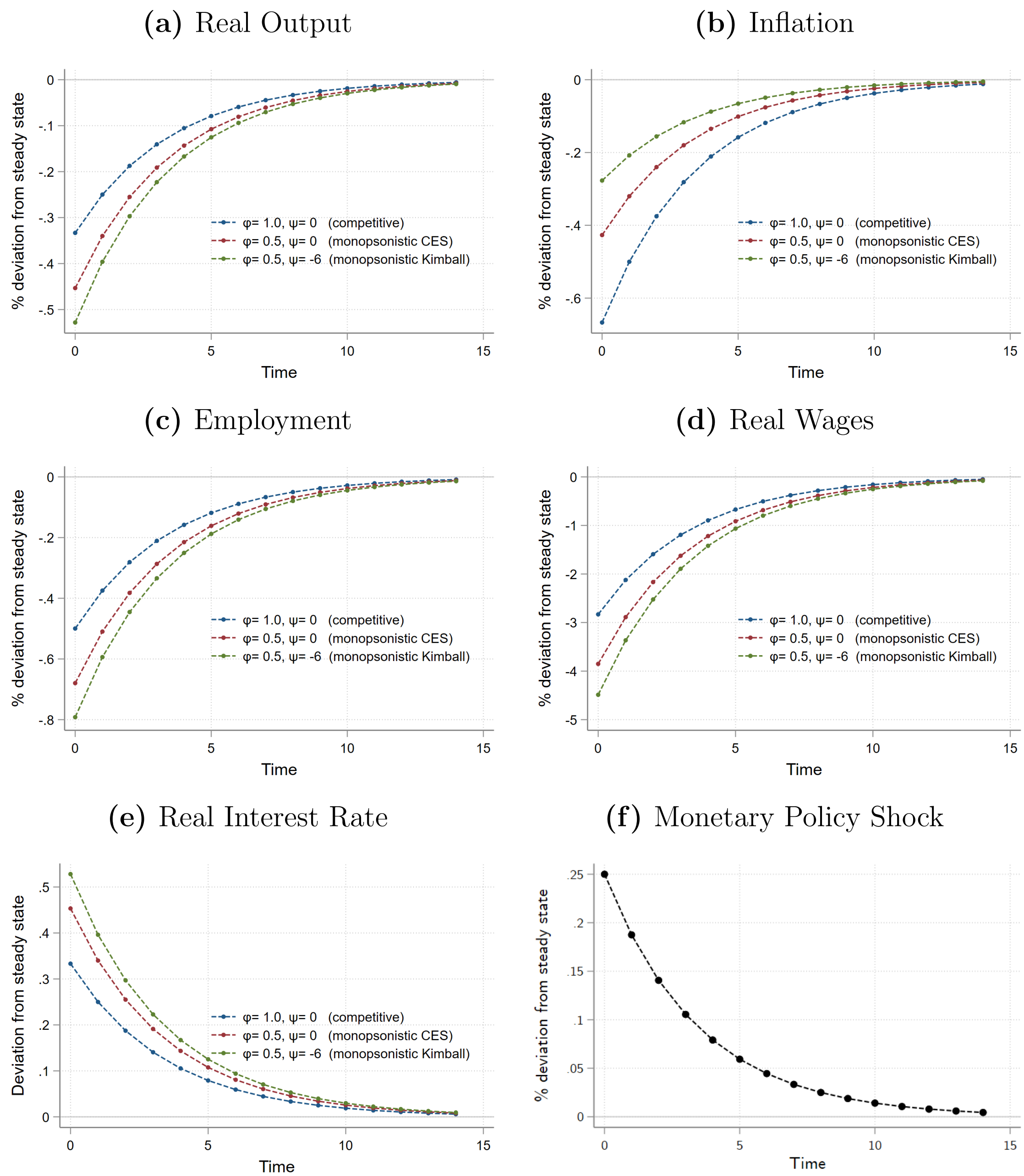
Notes: This figure documents heterogeneity in the baseline empirical design by local labor market concentration. Local labor market concentration is defined by computing a Herfindahl-Hirschman-type concentration, taking the squared shares of posting in each year. I assign each local labor market jl in year $y(t)$ a quintile corresponding to their concentration across locations, and partition my analysis sample according to quintiles. The instrument and endogenous regressor are interacted with indicator variables for each quantile. See Section 3.4 for more details.

Figure 10: IV Robustness: Leads and Lags of Outcome



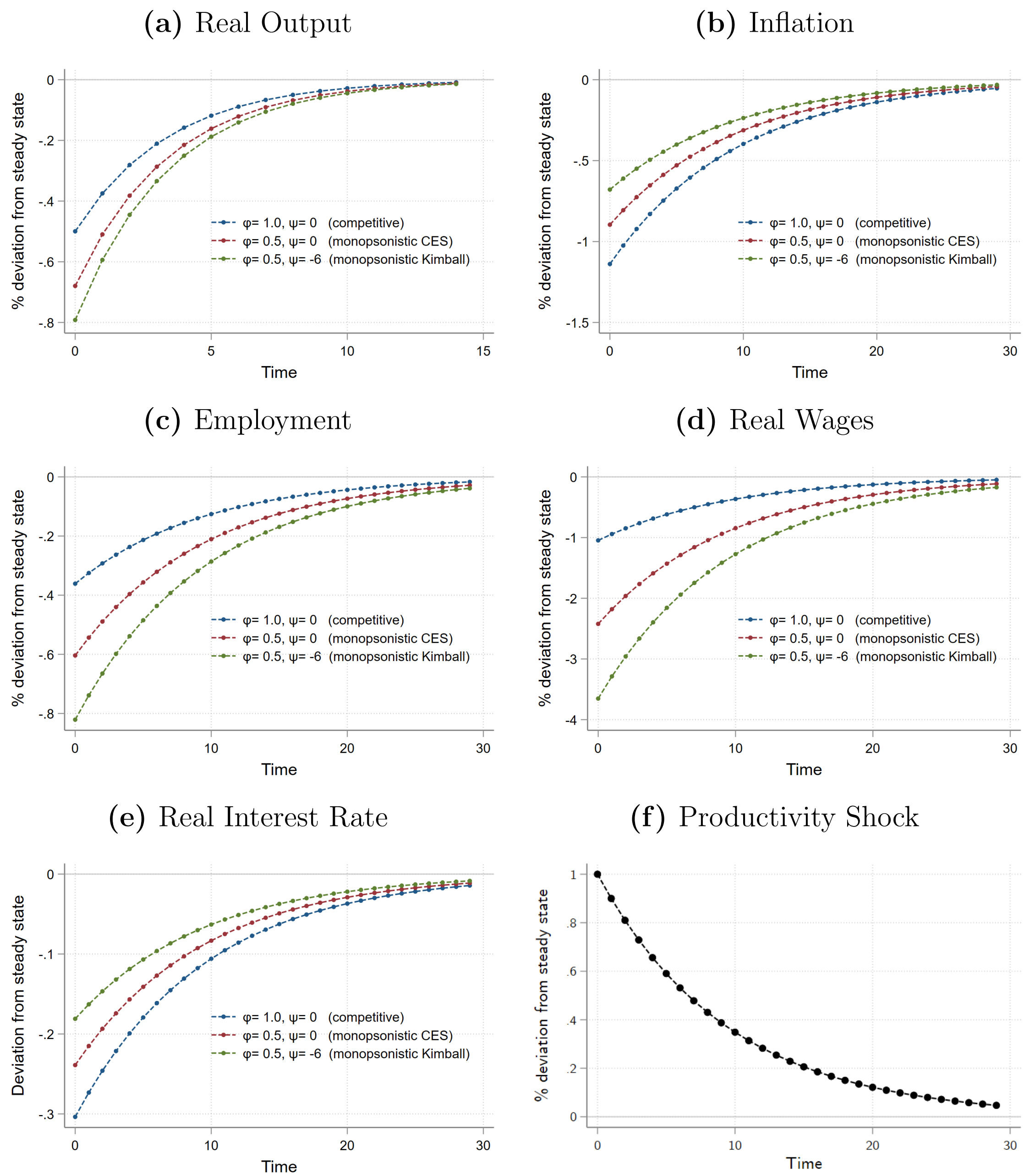
Notes: This figure estimates my baseline IV regression with leads and lags of the outcome variable. The baseline specification (column 2 of Table 2) is reported under the lead/lag of 0. Positive values on the x-axis correspond to leads, and negative numbers correspond to lags. See Section 3.4 for more details.

Figure 11: Impulse Responses to a Persistent Monetary Policy Shock



Notes: This figure documents the impulse responses to a persistent unanticipated monetary policy shock at $t = 0$ in the model described in Section 4. I consider three parameterizations corresponding to (1) competitive labor markets; (2) monopsonistic labor markets with CES labor supply; (3) monopsonistic labor markets with Kimball labor supply. Inflation and the real interest rate are annualized. See the main text for more details.

Figure 12: Impulse Responses to a Persistent Productivity Shock



Notes: This figure documents the impulse responses to a persistent unanticipated productivity shock at $t = 0$ in the model described in Section 4. I consider three parameterizations corresponding to (1) competitive labor markets; (2) monopsonistic labor markets with CES labor supply; (3) monopsonistic labor markets with Kimball labor supply. Inflation and the real interest rate are annualized. See the main text for more details.

A Data Appendix

This appendix provides additional information on the job postings data from Lightcast (formerly Burning Glass Technologies) that is used in the empirical component of this paper. This data was provided directly from Lightcast as a part of an institutional research subscription, and covers the period from January 1, 2010, to July 31, 2023.

A.1 Lightcast Processing and De-Duplication

The data that I start with has been processed from underlying scraped postings by Lightcast. This appendix section describes the de-duplication procedure that Lightcast has implemented on the data.

Lightcast scrapes job postings from tens of thousands of online sources every day. A company with a job opening may frequently post the same job in multiple sources. De-duplicating these postings across sources ensures that the same job opening is not represented multiple times purely due to the employer's use of multiple job posting platforms. Lightcast's procedure for de-duplicating is as follows. New job postings are scraped from all sources every day. New job postings are de-duplicated across sources by looking to see whether a given (job title, company, location) has been posted in other sources in a 60-day window. Job titles and locations are standardized with regular expressions, but otherwise reflect the finest level of detail in a given posting.

To illustrate this de-duplication procedure, suppose that Google posts a job opening for a full-stack software developer based in Mountain View, CA in March 1st. This posting is advertised on Google's official careers page (careers.google.com) and also on linkedin.com. If Google posts a *new* job opening - that is, a new job advertisement on any of the preceding sources or additional sources, at any point between March 1 and April 30, these will be regarded as duplicate postings. The number of duplicate postings associated with a given posting *is* available in the processed data available to me. About 80% of postings have at least 1 duplicate.

A natural disadvantage of this procedure, and of online job postings in general, is that a single online job posting can correspond to multiple job openings. In the prior example, a single job posting from Google might correspond to multiple vacancies. While information on the underlying number of vacancies is sometimes communicated on online job boards,[13], Lightcast has no way to reliably infer the underlying number

[13]For instance, Job Openings for Economists, which this author has recently become familiar with and which is scraped by Lightcast.

of vacancies. This is an important caveat to my analysis - and indeed, a broader caveat to any literature using similar job postings data to understand labor market dynamics - that I will defer for future research.

A.2 Panel Construction

My empirical analysis relies on a panel dataset that is constructed by cleaning the underlying posting-level data and aggregating the underlying (processed) Lightcast data within (company,occupation,location,month) cells. This appendix subsection describes this procedure in more detail.

I first clean the underlying posting-level data prior to aggregation. I drop all postings that correspond to internships, or that correspond to large job recruitment agencies. I remove these postings because they are not regular employment contracts and because the companies being represented by job recruiters are not often identified in job advertisements.

Next, I redefine company identifiers to account for transcription error in the underlying data. In plain terms, company identifiers in the processed lightcast data are assigned from unique company name strings. However, the company name strings sometimes exhibit transcription error: the same companay can be identified under multiple strings, either due to transcription errors (Lightcast web scrapers read characters incorrectly) or due to actual inconsistency in the underlying company titles across postings. For instance, a job posting for a store supervisor at Whole Foods might list the company as "Whole Foods", "Whole Foods Market", or "Whole Foods Market, Inc." Naturally, it is desirable to treat all three of these cases as the same underlying company.

This issue with the Lightcast data was first noted by Hazell et al. (2022), who develop a procedure to clean and agglomerate titles. In essence, their approach is to first apply a series of regular expressions to company name strings, and then to use a fuzzy matching procedure that agglomerates company names when a suitable string distance metric is sufficiently close to zero. I redefine unique numeric company identifiers based on these agglomerated company strings.

B Theory Appendix

This appendix provides additional information on the results from Sections 2, 3, and 4 of the main text.

B.1 Section 2 Derivations

This section derives some key results from the stylized model of Section 2.

B.1.1 Kimball Labor Supply Curves

The Lagrangian corresponding to the household's utility maximization problem can be written:

$$\mathcal{L} = U(C, L) + \lambda_0 \left[\int_0^1 w_i \ell_i di - C \right] - \lambda_1 \left[1 - \int_0^1 \Upsilon(\ell_i / L) di \right]$$

where λ_0 and λ_1 are the Lagrangian multipliers corresponding to the household's budget constraint and the Kimball labor supply aggregator, respectively.

The first-order conditions for this problem with respect to C, L, and ℓ_i are:

$$\begin{aligned} \partial \mathcal{L} / \partial C = 0 \quad &\Longrightarrow \quad U_C = \lambda_0 \\ \partial \mathcal{L} / \partial L = 0 \quad &\Longrightarrow \quad U_L L = \lambda_1 \int_0^1 \Upsilon'(\ell_i / L)(1/L) di \\ \partial \mathcal{L} / \partial \ell_i = 0 \quad &\Longrightarrow \quad L \lambda_0 \lambda_1^{-1} = w_i^{-1} \Upsilon'(\ell_i / L) \end{aligned}$$

My objective is to write the household's choice of firm-specific labor supply ℓ_i in terms of the wage w_i, model primitives, and the appropriate notion of an aggregate wage index W. Combining the first-order condition $\partial \mathcal{L} / \partial \ell_i = 0$ for any two firms i, j:

$$\frac{w_j}{w_i} = \frac{\Upsilon'(\ell_j / L)}{\Upsilon'(\ell_i / L)} \tag{B.1}$$

Multiplying both sides by $w_i \ell_j$, integrating both sides over $j \in [0, 1]$, and factoring out constants inside the integral (with respect to i) on the right-hand side yields:

$$\int_0^1 \ell_j w_j dj = \frac{w_i}{\Upsilon'(\ell_i / L)} \int_0^1 \Upsilon'(\ell_j / L) \ell_i di \tag{B.2}$$

Dividing both sides by L and solving for w_i:

$$w_i = \frac{\left(\int_0^1 w_j (\ell_j / L) dj \right) \Upsilon'(\ell_i / L)}{\int_0^1 \Upsilon'(\ell_j / L)(\ell_j / L) dj} \tag{B.3}$$

where I have also exploited the linearity of the integral operator to factor in L.

Next, I define the aggregate wage index W as:

$$W = \frac{\int_0^1 w_j(\ell_j/L)dj}{\int_0^1 \Upsilon(\ell_j/L)(\ell_j/L)dj} \tag{B.4}$$

Substituting (B.4) into (B.3) yields firm-specific relative labor supply in terms of the firm-specific relative wage, as desired:

$$\frac{w_i}{W} = \Upsilon'(\ell_i/L) \tag{B.5}$$

B.1.2 Proof of Proposition 1

This subsection proves Proposition 1 from the main text.

When $\psi = 0$, the optimal wage satisfies:

$$w_i^* = \mathcal{M}_i F_{\ell i} = \frac{(1-\alpha)A_i\ell_i^{-\alpha}}{1+1/\epsilon_i} \tag{B.6}$$

with $\epsilon_i = \partial \ln \ell_i / \partial \ln w_i^* = \phi/(1-\phi) = \epsilon$, where I suppress the subscript on the elasticity to note that it is constant and determined by ϕ.

Household labor supply to firm i with $\psi = 0$ given a wage w_i^* satisfies:

$$\ell_i = L\left(\frac{w_i^*}{W}\right)^{\frac{1-\omega}{\omega}} \tag{B.7}$$

where $\omega = \phi$ in this case when $\psi = 0$.

Substituting in household labor supply into the first-order condition for wages:

$$w_i^* = \frac{(1-\alpha)A_i L^\alpha \left(\frac{w_i^*}{W}\right)^{\alpha\frac{1-\omega}{\omega}}}{1+\phi/(1-\phi)} \tag{B.8}$$

Placing this in terms of the elasticity ϵ:

$$w_i^* = \frac{(1-\alpha)A_i L^\alpha \left(W/w_i^*\right)^{\alpha\epsilon}}{1+\epsilon} \tag{B.9}$$

Taking logs of both sides and solving for $\ln(w_i^*)$:

$$\ln(w_i^*) = \frac{1}{1+\alpha\epsilon}\Bigg[\ln(1-\alpha) + \alpha\ln(L) - \ln(1+\epsilon) + \alpha\epsilon\ln(W)\Bigg] \tag{B.10}$$

Defining $\kappa = \frac{\ln(1-\alpha)+\ln(A_i)+\alpha\ln(L)-\ln(1+\epsilon)}{1+\alpha\epsilon}$ and $\beta = \frac{\alpha\epsilon}{1+\alpha\epsilon}$ we have:

$$\ln(w_i^*) = \kappa + \beta\ln(W) + (1-\beta)\ln(A_i) \tag{B.11}$$

This completes the proof.

B.1.3 Proof of Proposition 2

This subsection proves Proposition 2 from the main text.

Let $\Pi(w_i, W)$ denote the real profit function of firm i. Purely for notational convenience, define $\ell(w_i/W) = L(\Upsilon')^{-1}(w_i/W) = \ell_i$, expressing firm-specific labor supply explicitly as a function of the relative wage. Real profits can be written as:

$$\Pi(w_i, W) = A_i \ell(w_i/W) - w_i \ell(w_i/W) \tag{B.12}$$

Note that the optimal wage w_i^* satisfies the first-order condition $\Pi_1(w_i^*, W) = 0$. Differentiating with respect to w_i^*, we have:

$$\frac{\partial w_i^*}{\partial W}\frac{W}{w_i^*} = \frac{\Pi_{12}}{-\Pi_{11}}\frac{W}{w_i^*} \tag{B.13}$$

Expressing Π_{12} in terms of ℓ, $\ell' = \partial \ell/\partial w_i$, $\ell'' = \partial^2 \ell/\partial w_i^2$ (suppressing arguments for notational expediency):

$$\Pi_{12} = \frac{\ell''}{W}\left[\frac{\ell}{\ell'}\frac{w_i}{W}\right] - \frac{\ell'}{W}\left[\frac{1}{W} + \frac{\ell}{\ell'}\right] \tag{B.14}$$

Defining $\epsilon_\ell = \partial \ln \ell/\partial \ln w_i^* = (\ell'/\ell)(w_i/W)$ and expressing $\partial \epsilon_\ell/\partial w_i^*$ in terms of ℓ:

$$\frac{\partial \epsilon_\ell}{\partial w_i^*} = \frac{1}{\ell W}\left[\frac{\ell''}{W}w_i + \frac{\ell'}{W}\left[1 - \frac{\ell'}{\ell}\right]w_i\right] \tag{B.15}$$

Substituting (C.9) and the definition of ϵ_ℓ into (C.8) and simplifying:

$$\Pi_{12} = \frac{\ell}{\epsilon_\ell}\frac{-\partial \epsilon_\ell}{\partial w_i}\frac{w_i^*}{W} \tag{B.16}$$

Substituting (C.10) into (C.7) and simplifying:

$$\frac{\partial w_i^*}{\partial W}\frac{W}{w_i^*} = \frac{\ell}{-\Pi_{11}}\frac{-\partial \epsilon_\ell/\partial w_i^*}{\epsilon_\ell} \tag{B.17}$$

which is the desired result.

B.2 Section 4 Model Derivations

This section walks through the derivation of material omitted from Section 4, describing a New Keynesian model with strategic complementarity.

B.2.1 Optimal Reset Price

This subsection walks through the derivation of the optimal reset price in period t, p_t^*. The key difference relative to a "standard" New Keynesian model is that the firm does not take the wage as given. Their output choice implicitly impacts the wage rate through the household's firm-specific (Kimball) labor supply curve.

Recall from the main text that the optimal reset price p_t^* maximizes the expected discounted value of future profits, which is given by (13). The first-order condition associated with this problem is:

$$E_t \sum_{k=0}^{\infty} \theta^k \left[Q_{t,t+k} y_{t+k|t} \left(p_t^* - \frac{\epsilon}{\epsilon - 1} \mathcal{C}'(y_{t+k|t}) \right) \right] = 0 \tag{B.18}$$

where, as with (13), $Q_{t,t+k}$ denotes the stochastic discount factor that is used to discount firm profits between t and $t+k$, $y_{t+k|t}$ denotes demand in period $t+k$ conditional on resetting price in period t, and $\mathcal{C}'$ denotes the (nominal) marginal cost function.

It will be useful to log-linearize the reset price first-order condition (B.18) about the steady state:

$$\hat{p}_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\hat{mc}_{t+k|t} \right] \tag{B.19}$$

where $mc_{t+k|t}$ denotes real marginal costs (deflating nominal marginal costs $C'(y_{t+k|t})$ by the price index) and $\mu = \ln(\epsilon/(1 - \epsilon)$ denotes the log gross goods markup.

The nominal cost and nominal marginal cost functions $\mathcal{C}(\cdot)$ and $\mathcal{C}(\cdot)$, depend on the production function and household labor supply. A key difference relative to a textbook New Keynesian model with competitive labor markets is that under monopsonistic competition, firms must pay a higher (lower) wage to hire more (less) labor. With log-convex labor supply, this effect is amplified by non-constant markdowns.

B.2.2 New Keynesian Phillips Curve

This section briefly describes the derivation of the log-linearized New Keynesian Phillips Curve.

The firm-specific labor supply curve in period $s \geq t$ for a firm that last reset in period t is:

$$w_{t+k|t} = W_{t+k} \left[(1 + \psi)(\ell_{t+k|t}/L_{t+k}) - \phi \right]^{(1-\omega)/\omega} \tag{B.20}$$

where W_s is the wage index in period s.

Recall from the reset price problem that the nominal marginal cost function is $C'(y_{t+k|t}) = w_{t+k|t}\ell_{t+k|t}$. Log-linearizing *real* marginal costs, we have:

$$\hat{mc}_{t+k|t} = \hat{w}_{t+k|t} - \hat{mpl}_{t+k|t} - \hat{P}_{t+k} \tag{B.21}$$

Log-linearizing firm-specific labor supply, we have (letting $\hat{x}$ denote log deviations from steady state):

$$\hat{w}_{t+k|t} = \hat{W}_{t+k} + \frac{\omega(1+\psi)}{1-\omega}\big(\hat{\ell}_{t+k|t} - \hat{L}_{t+k}\big) \tag{B.22}$$

Likewise, by log-linearizing firm-specific labor demand:

$$\hat{\ell}_{t+k|t} = (1-\alpha)^{-1}\big(-\epsilon\big(\hat{p}_t^* - \hat{P}_{t+k}\big)\hat{Y}_{t+k} - \hat{A}_{t+k}\big) \tag{B.23}$$

where $\hat{Y}$ denotes aggregate output (in log deviations).
Loglinearizing the marginal product $mpl_{t+k|t} \equiv \partial y_{t+k|t}/\partial \ell_{t+k|t}$ and substituting in log-linearized firm-specific labor demand:

$$\hat{mpl}_{t+k|t} = \hat{A}_{t+k} - \alpha(1-\alpha)^{-1}\big(-\epsilon\big(\hat{p}_t^* - \hat{P}_{t+k}\big)\hat{Y}_{t+k} - \hat{A}_{t+k}\big) \tag{B.24}$$

Log-linearizing aggregate labor demand (given common production technology and symmetric equilibrium):

$$\hat{L}_{t+k} = (1-\alpha)^{-1}\big(\hat{Y}_{t+k} - \hat{A}_{t+k}\big) \tag{B.25}$$

Taking all of these together and plugging them into (B.21):

$$\hat{mc}_{t+k|t} = \hat{mc}_{t+k} - \frac{\alpha + (1-\phi)(1-\psi)}{1-\alpha} \tag{B.26}$$

The remainder of this problem is standard: we can express the reset price $\hat{p}^*$ recursively. Given Calvo pricing frictions, the goods price index satisfies:

$$P_t = \left[\theta\big(P_{t-1}\big)^{1-\epsilon} + (1-\theta)\big(p_t^*\big)^{1-\epsilon}\right]^{1/(1-\epsilon)} \tag{B.27}$$

Loglinearizing this about the zero inflation steady state and writing it in terms of inflation pi_t, we get:

$$\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}) \tag{B.28}$$

Substituting in the recursive representation of the linearized reset price $\hat{p}_t^*$, we obtain the New Keynesian Phillips Curve, as desired.

B.3 Loglinearized Model

This describes the full system of log-linearized equations in my model. Suppressing subscripts, where all endogenous variables are expressed as log deviations from a zero-inflation steady state:

$\hat{X}_t = -\sigma^{-1}\left(\hat{i}_t - E_t[\hat{\pi}_{t+1}] - \hat{r}_t^n\right) + E_t[X_{t+1}]$	Dynamic IS equation
$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \hat{X}_t$	New Keynesian Phillips Curve
$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{X}_t + \hat{\nu}_t$	Taylor rule
$\hat{Y}_t = \hat{A}_t + (1-\alpha)\hat{L}_t$	Output
$\hat{r}_t^n = -\sigma \Psi_N (1-\rho_a)\hat{A}_t$	Natural interest rate
$\hat{X}_t = \hat{Y}_t - \Psi_N \hat{A}_t$	Output gap
$\hat{W}_t = \sigma \hat{C}_t + \eta^{-1}\hat{L}_t + \hat{P}_t$	Labor FOC
$\hat{C}_t = \hat{Y}_t$	Resource constraint
$\hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1}$	Inflation
$\hat{r}_t = \hat{i}_t - E_t[\hat{\pi}_{t+1}]$	Real interest rate (Fisher)
$\hat{A}_{t+1} = \rho_a \hat{A}_t + \epsilon_t^a$	AR(1) productivity process
$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \epsilon_t^\nu$	AR(1) monetary shock process

The composite parameters κ and Ψ_N are given by:

$$\Psi_N = \frac{1+\eta^{-1}}{\sigma(1-\alpha)+\eta^{-1}+\alpha} \qquad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{\sigma + \frac{1/\eta+\alpha}{1-\alpha}}{1+\epsilon\frac{(1/\phi-1)(1-\psi)+\alpha}{1-\alpha}}$$